

Often, a Lipschitz function is referred to as a function that satisfies the *Lipschitz condition*. Furthermore, if a function f has a Lipschitz constant $L \in (0, 1)$, then f is a *contraction*, sometimes called a *contractive function*. (See Definition 2.5.10.) Thus intuitively, contractive images for two x -coordinates are closer together than those x -coordinates. Next, observe that the inequality in Definition 4.4.10 can be written as

$$\left| \frac{f(x) - f(t)}{x - t} \right| \leq L \quad \text{provided that } x \neq t.$$

Therefore, if the slope of the line segments joining $(x, f(x))$ and $(t, f(t))$ is bounded, then the function f is a Lipschitz function. Compare this with Example 4.4.12.

THEOREM 4.4.11. *If a function is a Lipschitz function, then it is uniformly continuous.*

A proof of this result is straightforward and is left as an exercise. Notice that the converse of Theorem 4.4.11 is not true. See Exercise 8(a).