**Definition 5.1.2.** *(Local)* Suppose that a function $f : D \to \Re$ with $D \subseteq \Re$, $a$ is an accumulation point of $D$, and $a \in D$. The *derivative of* $f$ *at* $x = a$ is defined by

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

provided that this limit is finite. If this is the case, we say that $f$ *is differentiable at* $x = a$.

Observe that in Definition 5.1.2, $x$ may approach $a$ from both sides or only from one side, depending on the function's domain.

**Example 5.1.3.** Verify that the function $f : \Re \to \Re$, defined by $f(x) = x^2$, is differentiable at any real value $x = a$.

*Answer.* Since $x = a$ is an accumulation point of $\Re$, we will attempt to evaluate the limit given in Definition 5.1.2. So we write

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{x^2 - a^2}{x - a} = \lim_{x \to a} \frac{(x - a)(x + a)}{x - a} = \lim_{x \to a} (x + a) = 2a.$$

Since $2a$ is a finite value, we can write that $f'(a) = 2a$. □

[2] The concept of a tangent line was introduced by Fermat around 1630.