Example 5.1.4. We will show that the function f, defined by $f(x) = \sqrt[3]{x}$, does not have a derivative at $x = 0$ (i.e., f is not differentiable at $x = 0$). We attempt to evaluate

$$\lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}.$$

This limit is equal to

$$\lim_{x\to 0} \frac{\sqrt[3]{x}}{x} = \lim_{x\to 0} \frac{1}{\sqrt[3]{x^2}} = \lim_{t\to +\infty} \sqrt[3]{t^2} = +\infty.$$

Even though this limit exists, its value is not finite, hence $f'(0)$ does not exist. Note that the tangent line to this function f at $x = 0$ is vertical (see Figure 5.1.2). □

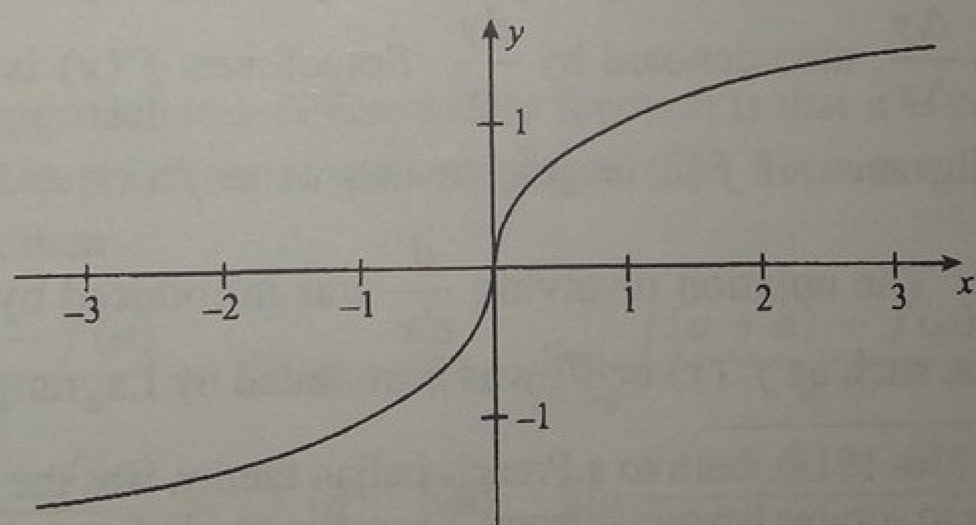

Figure 5.1.2