Definition 5.1.6. *(Global)* Suppose that $f : D \to \Re$ with $D \subseteq \Re$. If f is differentiable at every value in its domain D, then we say that f is *differentiable* (on D).

Note that in view of Example 5.1.3, $f(x) = x^2$ is a differentiable function because f has a derivative at every point $x = a$ in its domain. Observe that graph (f) has no vertical tangent lines and no sharp corners.

Let us now observe that if we set $h = x-a$ in Definition 5.1.2 (see Exercise 9 in Section 3.2), a derivative of f at $x = a$, if it exists, can be written as

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

We now replace a by x to obtain

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

This expression defines a new function f', the *derivative of* f, which is derived from f. Here, h can be replaced by Δx, which is a *change* (or *increment*) *in* x, depending on one's preference. In such a case, an *increment in* y, denoted by Δy, is given by $\Delta y = f(x + \Delta x) - f(x)$. Then $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$, also denoted by $\frac{dy}{dx}$. Sometimes $f'(x)$ is denoted by $D_x f$, f_x, $\frac{df}{dx}$, $\frac{d}{dx} f(x)$, etc. For instance, if $f(x) = x^2$, we can write $f'(x) = 2x$. This is equivalent to writing $\frac{d}{dx}(x^2) = 2x$. The notation involving $\frac{d}{dx}$ was introduced by Leibniz, whereas the notation involving primes, such as $f'(x)$ or y', was introduced by Lagrange.[3]. Newton used $\dot{y}$ to

[3]Joseph Louis Lagrange (1736–1813), born to a French-Italian family, was the youngest of eleven chil-