

denote  $y'$ . *Differentiating*, or *differentiation*, is the process of finding the derivative for a given function.

The concept of differentiability is stronger than that of continuity. We deduce this by considering the next result and recalling Examples 5.1.4 and 5.1.5, whose functions were continuous at  $x = 0$  but not differentiable there.

**THEOREM 5.1.7.** *If a function  $f$  is differentiable at a point  $x = a$ , then  $f$  must be continuous at  $x = a$ .*

*Proof.* Since  $x = a$  is an accumulation point of  $f$ , showing that  $f$  is continuous at  $x = a$  is equivalent to showing that the  $\lim_{x \rightarrow a} f(x) = f(a)$ , which, in turn, is equivalent to showing that the  $\lim_{x \rightarrow a} [f(x) - f(a)] = 0$ . Why? Thus, we write

$$\begin{aligned} \lim_{x \rightarrow a} [f(x) - f(a)] &= \lim_{x \rightarrow a} \left\{ [f(x) - f(a)] \left( \frac{x - a}{x - a} \right) \right\} \\ &= \lim_{x \rightarrow a} \left[ \frac{f(x) - f(a)}{x - a} (x - a) \right] \\ &= \left[ \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \right] \left[ \lim_{x \rightarrow a} (x - a) \right] = f'(a) \cdot 0 = 0. \end{aligned}$$

Note here that  $f'(a)$  is a finite number. Is that important? Why? This completes the proof.  $\square$