

5.2 Properties of Differentiable Functions

In this section we present a number of properties of differentiable functions, which we will refer to throughout the rest of the book. We start with the next theorem, which presents some of the facts about the sum, product, and quotient of differentiable functions.

THEOREM 5.2.1. *Suppose that functions $f, g : D \rightarrow \mathfrak{R}$ with $D \subseteq \mathfrak{R}$ are differentiable at $x = a$. Then $f \pm g$, fg , $\frac{f}{g}$ if $g(a) \neq 0$ are also differentiable at $x = a$ and*

$$(a) \quad (f \pm g)'(a) = f'(a) \pm g'(a).$$

$$(b) \quad (fg)'(a) = f(a)g'(a) + g(a)f'(a).$$

$$(c) \quad \left(\frac{f}{g}\right)'(a) = \frac{g(a)f'(a) - f(a)g'(a)}{[g(a)]^2}, \text{ provided that } g(a) \neq 0.$$

Parts (b) and (c) are known as the *product rule* and *quotient rule*, respectively.