Proof. Only the proof of part (b) will be given here. Parts (a) and (c) are left to the reader in Exercise 2. By performing a standard trick of adding "zero," we write

$$\begin{aligned}
\lim_{x\to a} \frac{(fg)(x) - (fg)(a)}{x-a} &= \lim_{x\to a} \frac{f(x)g(x) - f(a)g(x) + f(a)g(x) - f(a)g(a)}{x-a} \\
&= \lim_{x\to a} \left[g(x)\frac{f(x) - f(a)}{x-a} + f(a)\frac{g(x) - g(a)}{x-a} \right] \\
&= \left[\lim_{x\to a} g(x) \right] \left[\lim_{x\to a} \frac{f(x) - f(a)}{x-a} \right] + f(a) \left[\lim_{x\to a} \frac{g(x) - g(a)}{x-a} \right] \\
&= g(a)f'(a) + f(a)g'(a)
\end{aligned}$$

because f and g are differentiable and g is continuous at $x = a$ (Theorem 5.1.7). Thus, since this limit is finite, we have that $(fg)'(a) = g(a)f'(a) + f(a)g'(a)$. □