

The sum and product rule can be generalized to any finite number of functions.

COROLLARY 5.2.2. *If functions f_i , $i = 1, 2, \dots, n$, are differentiable on $D \subseteq \mathfrak{R}$, then so are functions $f_1 + f_2 + \dots + f_n$ and $f_1 \cdot f_2 \cdot \dots \cdot f_n$; and*

$$(a) \quad (f_1 + f_2 + \dots + f_n)'(x) = f_1'(x) + f_2'(x) + \dots + f_n'(x).$$

$$(b) \quad (f_1 f_2 \dots f_n)'(x) = f_1'(x) f_2(x) f_3(x) \dots f_n(x) + f_1(x) f_2'(x) f_3(x) \dots f_n(x) + \dots + f_1(x) f_2(x) f_3(x) \dots f_n'(x).$$

If $f_i = f$ for all $i = 1, 2, \dots, n$, then part (b) of Corollary 5.2.2 can be rewritten as

$$(f^n)'(x) = n f^{n-1}(x) f'(x).$$