Example 5.4.9. Use Taylor's theorem to approximate $\sin 3°$ to four-decimal-place accuracy; that is, the magnitude of the error is less than 0.5×10^{-4}.

Answer. Let us expand the function $f(x) = \sin x$ about $x = 0$, since 0 is the closest point to $3° = \frac{\pi}{60}$ radian, whose sine and cosine values are easily evaluated. Thus, we have

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + R_n(x),$$

where $R_n(x)$ involves $f^{(n+1)}(c)$, which in the case of $\sin x$ is $\pm \sin c$ or $\pm \cos c$ and the term $\frac{1}{(n+1)!} x^{n+1}$. We now replace x by $\frac{\pi}{60}$ and look for the value of n so that