

$\left| R_n \left(\frac{\pi}{60} \right) \right| < 0.5 \times 10^{-4}$. Note that if we expand $\sin x$ about a point more distant to 3° than 0, then the value of n in a related Taylor polynomial will be greater.

Thus, since $|f^{(n+1)}(c)| \leq 1$, we can write

$$\left| R_n \left(\frac{\pi}{60} \right) \right| \leq \left(\frac{\pi}{60} \right)^{n+1} \frac{1}{(n+1)!}.$$

Since this value is less than 0.5×10^{-4} when $n = 3$, we need only to compute

$$\sin 3^\circ = \sin \frac{\pi}{60} \approx \frac{\pi}{60} - \frac{\left(\frac{\pi}{60}\right)^3}{3!} \approx 0.0523.$$

Note that $n = 300$ will also make $\left| R_n \left(\frac{\pi}{60} \right) \right| < 0.5 \times 10^{-4}$, but it will require evaluating many more terms in $p_n \left(\frac{\pi}{60} \right)$. □