($\Leftarrow$) The function f, differentiable on I, implies that for any $x_0 \in I$, we have that $\lim_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$. The function f, Lipschitz on I, implies that $|f(x_1) - f(x_2)| \leq L|x_1 - x_2|$ for any $x_1, x_2 \in I$. This can be written as $\left|\dfrac{f(x) - f(x_0)}{x - x_0}\right| \leq L$ if we pick $x_1 = x$, $x_2 = x_0$ and $x \neq x_0$. Thus, $|f'(x_0)| \leq L$. Why? Hence f' is bounded on I. The proof is complete. □