Extema problems from calculus are no doubt familiar to the reader. Most problems of this type lead to a horizontal tangent at the extremum point (i.e., when the derivative is 0). This is not quite accurate. For example, the function $f(x) = x$ on $[0, 1]$ has a maximum at $x = 1$ and a minimum at $x = 0$. However, $f'(0) = f'(1) = 1$. Thus, the statement about the existence of a horizontal tangent at a maximum or minimum point is wrong. This problem can be corrected by means of the next theorem and some discussion and exercises that follow.

THEOREM 5.2.5. *Suppose that $f : D \to \Re$ has a relative extremum at $c \in (a, b) \subseteq D$. If f is differentiable at $x = c$, then $f'(c) = 0$.*

Proof. We prove the theorem only for the case where f has a relative maximum at $x = c$. Thus, since f has a relative maximum at $x = c$, there exists $\delta > 0$ such that $f(x) \leq f(c)$ for all $|x - c| < \delta$. Thus, for any $h \in (-\delta, \delta)$, we have $f(c + h) - f(c) \leq 0$. Therefore, we can write

$$\frac{f(c+h) - f(c)}{h} \text{ is } \begin{cases} \leq 0 & \text{if } h > 0 \\ \geq 0 & \text{if } h < 0, \end{cases}$$