with $h \in (-\delta, \delta)$. Hence, $\lim_{h\to 0^+} \dfrac{f(c+h)-f(c)}{h} \leq 0$ and $\lim_{h\to 0^-} \dfrac{f(c+h)-f(c)}{h} \geq 0$. But since f is differentiable at $x = c$, both of these limits must give the same value. Why? Therefore, $f'(c) = \lim_{h\to 0} \dfrac{f(c+h)-f(c)}{h} = 0$. □

Find an example of a function that has a relative extremum at $c \in (a, b)$ for which $f'(c) \neq 0$. The expression $f'(c) \neq 0$, in this case, does not mean that $f'(c)$ exists, but its value is different from 0 because the Theorem 5.2.5 would be contradicted. Why? See Exercise 7. How about conversely: If f is differentiable on (a, b) with $c \in (a, b)$ such that $f'(c) = 0$, does f necessarily have a relative extremum at $x = c$? Explain. The points $x = c$ for which $f'(c) = 0$ are called *stationary points*, that is, where tangent lines are horizontal. Next, we repeat part (b) of Definition 1.2.15. Compare it to Definition 5.2.4.

Definition 5.2.6. Let $f : D \to \Re$ with $D \subseteq \Re$. A function f is said to have an *absolute (global) maximum* (or simply *maximum*) at a point $x = c \in D$ if and only if $f(c) \geq f(x)$ for all $x \in D$.