Remark 5.3.2.

(a) Depending on the function, there could be more than one point $c \in (a, b)$ for which the derivative vanishes.

(b) Each condition in Rolle's theorem is essential. See Exercise 1.

(c) Intuitively speaking, if f starts and ends at the same height, when we connect those points in a "smooth" fashion, we see that there must be a point between a and b at which the tangent line is horizontal. Examine Figure 5.3.1, where we indicated two such cs and called them c_1 and c_2. Obviously, these horizontal tangent lines are parallel to the horizontal line through points $(a, f(a))$ and $(b, f(b))$. This trivial idea will show a connection to the mean value theorem, which follows. □

---

[6]Michel Rolle (1652–1719), a French mathematician, is best known for his book on the algebra of equations entitled *Traité d'algàbre*, published in 1690. In *Traité d'algàbre*, Rolle firmly established the notation $\sqrt[n]{a}$. In 1846, Giusto Bellavitis (1803–1880), an Italian mathematician who made significant contributions to algebraic and descriptive geometry, gave Theorem 5.3.1 the name *Rolle's theorem* as we know it today. Rolle also proved a polynomial version of Rolle's theorem.