Note that g satisfies all the hypotheses of Rolle's theorem. Why? Hence, by Rolle's theorem, there exists $c \in (a, b)$ such that $g'(c) = 0$. Therefore,

$$f'(c) = \frac{f(b) - f(a)}{b - a},$$

which completes the proof. □

Observe that the function g in the proof of the mean value theorem is not unique. Another commonly used g is

$$g(x) = f(x) - f(a) - \frac{f(b) - f(a)}{b - a}(x - a).$$

Rolle's theorem and the mean value theorem are important tools in proving results in various areas of mathematics and physics. Theorems are often proven using the mean value theorem, even though they do not mention derivatives. This is pointed out in many exercises and examples and in Section 5.4. The mean value theorem is not true in the context of complex numbers.