

Example 5.3.6. Use the mean value theorem to find upper and lower bounds of $\sqrt{51}$.

Answer. Apply the mean value theorem to the function $f(x) = \sqrt{x}$ on an interval, say $[49, 51]$ to obtain

$$\frac{f(51) - f(49)}{51 - 49} = \frac{1}{2\sqrt{c}}$$

for some $c \in (49, 51)$. Thus, because we have $\sqrt{51} - \sqrt{49} = \frac{1}{\sqrt{c}}$ with $49 < c < 51 < 64$

(why 64?), we can write $\frac{1}{8} < \frac{1}{\sqrt{c}} < \frac{1}{7}$, and therefore, $7\frac{1}{8} < \sqrt{51} < 7\frac{1}{7}$ (why?), so $\frac{57}{8} <$

$$\sqrt{51} < \frac{50}{7}.$$

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