Before we close this section, let us clarify one point that is often interpreted incorrectly.

THEOREM 5.3.9. *Suppose that the function f is defined on a neighborhood of $x = a$ with $f'(a) > 0$. Then there exists $\delta > 0$ such that $f(a) < f(x_1)$ for some $x_1 \in (a, a + \delta)$ and such that $f(x_2) < f(a)$ for some $x_2 \in (a - \delta, a)$.*

Remark 5.3.10. Theorem 5.3.9 does *not* say that if $f'(a) > 0$, then there exists a neighborhood of a on which f is increasing. The fact that $f'(x) > 0$ at only one point $x = a$ is not enough to guarantee that f must be increasing on some neighborhood of a. See Example 5.3.11. For the function f to be increasing on an interval I, $f'(x)$ must be nonnegative for all x in I. See Exercise 6. Also note that Theorem 5.3.9 can be restated with reverse inequalities. Proof of Theorem 5.3.9 is left as Exercise 26. □