

Example 5.3.11. Consider the function⁷ f , defined by

$$f(x) = \begin{cases} x + 2x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Verify that

$$f'(x) = \begin{cases} 1 + 4x \sin \frac{1}{x} - 2 \cos \frac{1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0. \end{cases}$$

Moreover, observe that even though $f'(0) = 1 > 0$, for any $\delta > 0$, there exists x_1 and x_2 in the interval $(-\delta, \delta)$ for which $f'(x_1) > 0$ and $f'(x_2) < 0$. Therefore, f is not monotonic in any neighborhood of $x = 0$. See Figure 5.3.3.

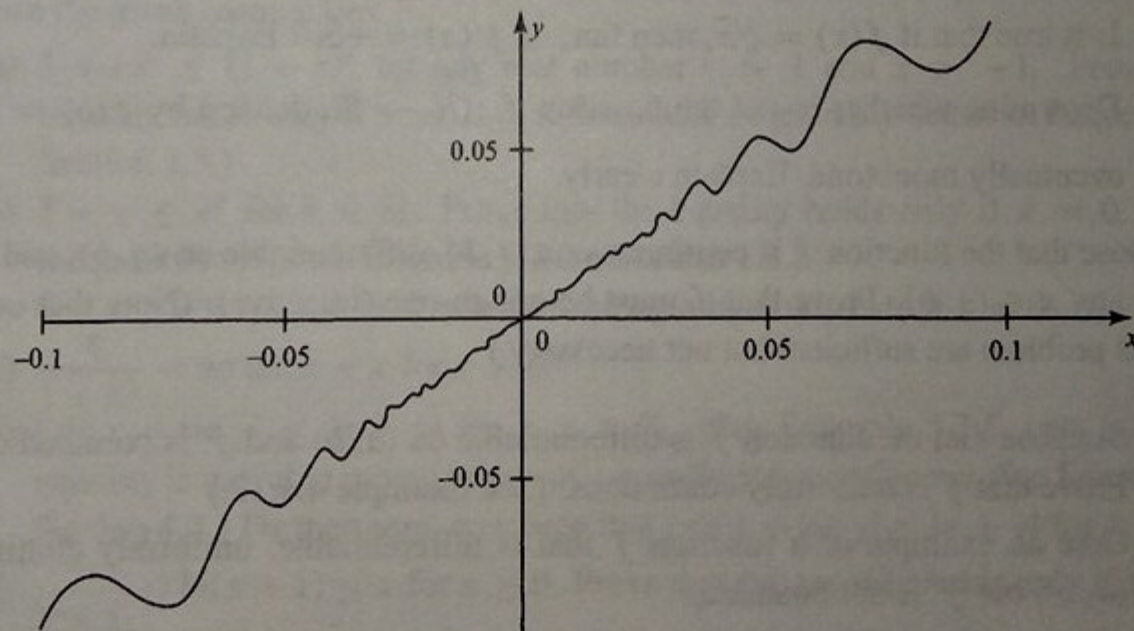


Figure 5.3.3