5.4 Higher-Order Derivatives

Concavity, n times continuous differentiability, uniform differentiability, multiplicity of roots, and Taylor's theorem are among topics covered in this section. These ideas come up in differential equations, numerical analysis, and many other areas of mathematics. In analysis one cannot live without thorough knowledge of these topics.

Consider a differentiable function f. If the derivative function f' has a derivative at $x = a$, then that number is the *second derivative* of f at $x = a$ and is denoted by $f''(a)$. The *third*, *fourth*, and *fifth* through nth *derivatives* are defined similarly. Thus, we can write

$$f''(a) = \lim_{x \to a} \frac{f'(x) - f'(a)}{x - a} = \lim_{h \to 0} \frac{f'(a+h) - f'(a)}{h} = \lim_{n \to \infty} \frac{f'(a+h_n) - f'(a)}{h_n},$$

where $\{h_n\}$ is any sequence converging to 0 with $a + h_n \in \text{domain}\,(f')$. However, if we can differentiate a function f over and over on its domain, then we will obtain the *first*, *second*, and

[10] Jean Gaston Darboux (1842–1917), a French mathematician, excelled in differential geometry and did work with the intermediate value property.