This, however, is not always possible.

Second-order derivatives are useful in studying concavity of functions, acceleration in physics, road planning in engineering, and a great many other areas. For a quick review, consider the following definition.

**Definition 5.4.1.** If $f$ is a differentiable function on an interval $(a, b)$ and $f'$ is strictly increasing on $(a, b)$, then $f$ is *concave up* on $(a, b)$. *Concavity downward* is defined similarly.

Intuitively, this definition says that a differentiable function $f$ is concave up on $(a, b)$ if and only if it lies above its tangent lines. Recall from Section 5.3, Exercise 6, that if the derivative of a function is positive, then that function is strictly increasing. Therefore, if the function $f$ is twice differentiable and $f'' > 0$, then $f'$ must be strictly increasing. Thus, according to Definition 5.4.1, $f$ must be concave up. This proves the next theorem informally. Figure 5.4.1 shows a function $f$ that is twice differentiable, together with several tangent lines whose slope, from left to right, is getting larger and larger.

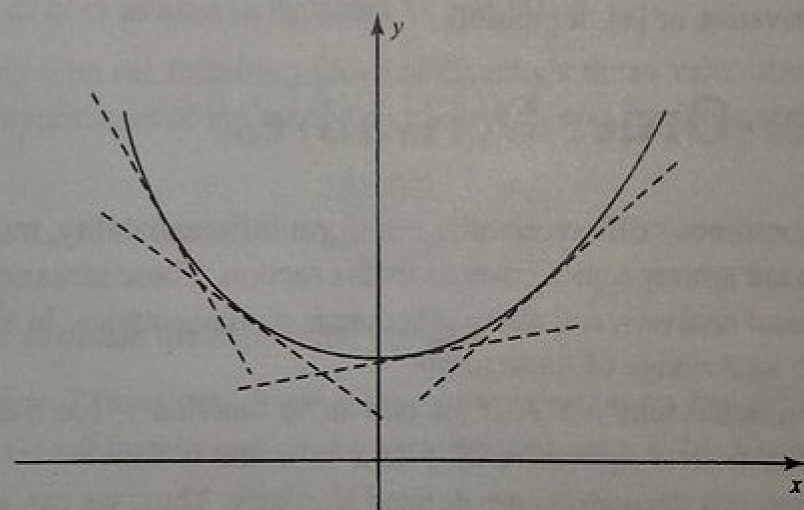


**Figure 5.4.1**

**THEOREM 5.4.2.** *If a function $f$ satisfies $f''(x) > 0$ for all $x \in (a, b)$, then $f$ must be concave up on $(a, b)$. Similarly, $f''(x) < 0$ on $(a, b)$ implies that $f$ is concave down on $(a, b)$.*