

Homework #2 - Independence

Exercise 1. Consider the experiment which consists in throwing a fair die. Define X the random variable which is the number obtained. Define the random variable Y which equals 1 if X is even, and equals 0 if X is odd.

1. Find the distribution of the couple (X, Y) .
2. Are X and Y independent?

Exercise 2. Consider the experiment which consists in throwing two fair dice. Consider the events

A = the first die brings an even number.

B = the second die brings an odd number.

C = both die brings a number of same parity.

1. Prove that the events A, B, C are pairwise independent.
2. Are A, B, C independent?

Exercise 3. Let X and Y be continuous random variables with density f_X and f_Y respectively, and joint density $f_{X,Y}$. Assume that f_X, f_Y and $f_{X,Y}$ are continuous functions.

1. Prove that $F'_X(x) = f_X(x)$ for all $x \in \mathbb{R}$.
2. Prove that X and Y are independent if and only if for all $x, y \in \mathbb{R}$, $f_{(X,Y)}(x, y) = f_X(x)f_Y(y)$.

Exercise 4. Denote $\Omega = \{A, B, C, D, E\}$, and consider the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where \mathbb{P} is the uniform distribution on Ω . Define the random variables

$$X(A) = 10, X(B) = 5, X(C) = 10, X(D) = 5, X(E) = 10,$$

$$Y(A) = 10, Y(B) = 10, Y(C) = 10, Y(D) = 5, Y(E) = 5.$$

1. Compute $\mathbb{P}(X = 5)$ and $\mathbb{P}(X = 10)$.
2. Same question for Y .
3. Are X and Y independent?

Exercise 5.

1. Let A be an event independent of itself. What is the probability of A ?
2. Let A be an event of probability 0 or 1. Show that A is independent of any events.