University of Florida **MAP 6472**

Homework #2

Exercise 1.

1. Let $X, Y\colon (\Omega, \mathcal{F}) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$ be random variables. Prove that $X+Y, XY, |X|$ are random variables as well.

2. Let $\{X_n\}$ be a sequence of random variables. Show that $\liminf X_n$ and $\limsup X_n$ are random variables.

Exercise 2. On the measurable space $(\Omega, \{\emptyset, \Omega\})$, what necessary and sufficient condition for $X\colon (\Omega, \{\emptyset, \Omega\}) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$ to be measurable?

Exercise 3. Let $X\colon (\Omega, \mathcal{F}) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$ be a random variable. Prove that the smallest sigma-algebra generated by X is

$$\sigma(X) = X^{-1}(\mathcal{B}(\mathbb{R})).$$

Exercise 4. Show that if $X\colon (\Omega, \mathcal{F}) \to (S, \mathcal{S})$ and $f\colon (S, \mathcal{S}) \to (T, \mathcal{T})$ are measurable functions, then $f(X)$ is measurable map from $(\Omega, \mathcal{F})$ to $(T, \mathcal{T})$.

Exercise 5. Let $F\colon \mathbb{R} \to [0,1]$ be a function satisfying the properties of a CDF (Cumulative Distribution Function). Show that there is a random variables X on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that the CDF of X is F.

Exercise 6. Show that a CDF has at most countably many discontinuities.

Exercise 7. Compute the CDF of the following distributions:

1. Bernoulli distribution,

2. uniform distribution on $(0, 1)$,

3. exponential distribution,

4. normal distribution.

Exercise 8. Let $n \geq 2$. Consider the discrete random variable X taking values on $\{2, \ldots, n\}$ with probabilities

$$\mathbb{P}(X = i) = \frac{C_n(i-1)}{n}, \quad i = 2, \ldots, n.$$

1. Find the value of C_n.

2. Compute $\mathbb{E}[X]$.