New connections between the entropy power inequality and geometric inequalities

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***Abstract*—The entropy power inequality (EPI) has a fundamental role in Information Theory, and has deep connections with famous geometric inequalities. In particular, it is often compared to the Brunn-Minkowski inequality in convex geometry. In this article, we further strengthen the relationships between the EPI and geometric inequalities. Specifically, we establish an equivalence between a strong form of reverse EPI and the hyperplane conjecture, which is a long-standing conjecture in high-dimensional convex geometry. We also provide a simple proof of the hyperplane conjecture for a certain class of distributions, as a straightforward consequence of the EPI.**

Keywords. Entropy power inequality, reverse entropy power inequality, hyperplane conjecture.

I. Introduction

Since the pioneering work of Costa and Cover [9], and Dembo, Cover and Thomas [13], several relationships have been developed between information theory and convex geometry.

We explore in this article an intimate connection between the entropy power inequality (EPI), fundamental in information theory, and the hyperplane conjecture, a major unsolved problem in high-dimensional geometry.

Let us recall that the entropy power inequality of Shannon [26] states that for all independent continuous random vectors X, Y in $\mathbb{R}^n$,

$$N(X+Y) \geq N(X) + N(Y), \tag{1}$$

where $N(X) \triangleq \frac{1}{2\pi e} e^{\frac{2}{n}h(X)}$ denotes the entropy power of X. Furthermore, the hyperplane conjecture, raised by Bourgain [8], can be stated as follows.

Conjecture 1 (Hyperplane Conjecture [8])**.** *There exists a universal constant $c > 0$ such that for every $n \geq 1$, for every convex body $K \subset \mathbb{R}^n$ of volume 1, there exists a hyperplane H such that*

$$\mathrm{Vol}_{n-1}(K \cap H) \geq c, \tag{2}$$

where $\mathrm{Vol}_{n-1}(\cdot)$ denotes the $(n-1)$-dimensional volume.

There are several equivalent formulations of the hyperplane conjecture, mainly expressed in geometric or probabilistic language. One of particular interest is an information-theoretic formulation developed by Bobkov and Madiman [4]. Let us recall that a random vector X in $\mathbb{R}^n$ is isotropic if X is centered and if $K_X = \mathbb{E}\left[XX^T\right]$, the covariance matrix of X, is the identity matrix I_n, and that X is log-concave if $X \sim f_X$ with $\log(f_X)$ concave.

Conjecture 2 (Entropic Hyperplane Conjecture [4])**.** *There exists a universal constant $c > 0$ such that for every $n \geq 1$, for every isotropic log-concave random vector X in $\mathbb{R}^n$,*

$$D(X||Z) \leq cn, \tag{3}$$

where $D(X||Z)$ denotes the relative entropy between X and the standard Gaussian $Z \sim \mathcal{N}(0, I_n)$.

Although the isotropicity condition is not assumed in the formulation of [4], the above formulation of Conjecture 2 is actually equivalent. This is because the quantity

$$D(X||Z_X) = h(Z_X) - h(X), \tag{4}$$

where $Z_X \sim \mathcal{N}(0, K_X)$, is affine invariant, and we can always find for arbitrary X an affine transformation T such that $T(X)$ is isotropic. Hence, we can assume without loss of generality that we work with isotropic random vectors.

Let us briefly explain the equivalence between Conjectures 1 and 2. The isotropic constant of a random vector $X \sim f_X$ in $\mathbb{R}^n$ is defined as

$$L_X^2 \triangleq \|f_X\|_\infty^{\frac{2}{n}} |K_X|^{\frac{1}{n}}, \tag{5}$$

where $\|f_X\|_\infty$ denotes the essential supremum of f_X, and $|K_X|$ denotes the determinant of K_X. A convex body K is isotropic if K is centered, $\mathrm{Vol}(K) = 1$,

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and its covariance matrix $\left(\int_K x_i x_j \, dx\right)_{1\leq i,j\leq n}$ is a multiple of the identity. Hensley [15] proved that there exist universal constants $d_1, d_2 > 0$ such that for any isotropic convex body K, for any hyperplane H passing through the origin,

$$d_1 \leq L_K^2 \mathrm{Vol}_{n-1}(K \cap H) \leq d_2. \tag{6}$$

Here, $L_K \triangleq L_X$ with X uniformly distributed on K. Hence, lower bounding $\mathrm{Vol}_{n-1}(K \cap H)$ by a universal constant (Conjecture 1) is equivalent to upper bounding L_K by a universal constant. Ball [1] proved that upper bounding L_K by a universal constant for any isotropic convex body K is equivalent to upper bounding L_X by a universal constant for any isotropic log-concave random vector X. Finally, Bobkov and Madiman [4] proved that the Rényi entropies of a log-concave random vector X in $\mathbb{R}^n$ are comparable,

$$h_\infty(X) \leq h(X) \leq h_\infty(X) + n, \tag{7}$$

where $h_\infty(X) \triangleq -\log\left(\|f_X\|_\infty\right)$ is the ∞-Rényi entropy of X. Using (7), they [4] showed that upper bounding L_X by a universal constant for any isotropic log-concave random vector X is equivalent to Conjecture 2.

Information-theoretic formulations have seen an increasing interest over the past few years, mainly due to the simplicity of the formulations, and due to the simplicity of the ensuing proofs. For example, an information-theoretic argument easily yields the monotonicity of entropy (see [28], [22], [10]), properties of the maximal correlation between sums of random variables (see [17]), and important functional inequalities in mathematics (see [27]). Furthermore, the validity of the hyperplane conjecture has several applications to information theory. For example, bounds can be obtained for the entropy rate of log-concave random processes (see [5]), as well as for the capacity and rate-distortion function under log-concavity assumptions (see [24]).

The goal of this article is to develop a new information-theoretic formulation of the hyperplane conjecture (Section II), and to show that the EPI immediately implies the hyperplane conjecture for a class of random vectors more general than the class of (c_1, c_2)-regular distributions introduced by Polyanskiy and Wu [25], hence simplifying and extending a result of Dörpinghaus [14], who implicitly showed that Conjecture 2 holds for isotropic log-concave (c_1, c_2)-regular distributions, as long as $c_1 \leq c$ and $c_2 \leq c\sqrt{n}$ for some universal constant $c > 0$ (Sections III and IV).

II. Equivalence between reverse entropy power inequality and hyperplane conjecture

In this section, we establish a new information-theoretic formulation of the hyperplane conjecture as a reverse form of the EPI. We introduce the following conjecture.

Conjecture 3. *There exists a universal constant $c' > 0$ such that for every $n \geq 1$, for every $k \geq 1$, for i.i.d. isotropic log-concave random vectors $X_1, \ldots, X_k$ in $\mathbb{R}^n$,*

$$\begin{aligned} N(X_1 + \cdots + X_k) &\leq c' \left(N(X_1) + \cdots + N(X_k)\right) \\ &= c' k N(X_1). \end{aligned} \tag{8}$$

Theorem 1. *Conjectures 2 and 3 are equivalent.*

Proof. Assume first that Conjecture 3 is true. Let $X_1, \ldots, X_k$ be i.i.d. isotropic log-concave random vectors. Then,

$$N(X_1 + \cdots + X_k) \leq c' k N(X_1). \tag{9}$$

It follows that

$$N\left(\frac{X_1 + \cdots + X_k}{\sqrt{k}}\right) \leq c' N(X_1). \tag{10}$$

By letting k go to $+\infty$, and using the entropic central limit theorem developed by Barron [3] (valid in any dimension, see [16]), we obtain

$$N(Z) \leq c' N(X_1), \tag{11}$$

where Z is standard Gaussian. Hence,

$$1 \leq c' N(X_1). \tag{12}$$

To conclude, we note that

$$\begin{aligned} 1 \leq c' N(X_1) &\iff \log(2\pi e) \leq \log(c') + \frac{2}{n} h(X_1) \\ &\iff D(X_1 \| Z) \leq cn, \end{aligned} \tag{13}$$

where $c = \frac{\log(c')}{2}$, which is the hyperplane conjecture in the equivalent version of Conjecture 2.

Now, assume that Conjecture 2 is true. Let $k \geq 1$, and let $X_1, \ldots, X_k$ be i.i.d. isotropic log-concave random vectors. Since X_i's are i.i.d. and isotropic, we have

$$|K_{X_1 + \cdots + X_k}|^{\frac{1}{n}} = |K_{X_1} + \cdots + K_{X_k}|^{\frac{1}{n}} = k. \tag{14}$$

It is well known that Gaussian distributions maximize entropy under covariance matrix constraint (see, e.g., [16]), hence we have the following upper bound for any X with finite covariance matrix K_X,

$$h(X) \leq h(Z_X) = \frac{n}{2} \log(2\pi e |K_X|^{\frac{1}{n}}). \tag{15}$$

Thus, using (14),

$$h(X_1+\cdots+X_k) \le \frac{n}{2}\log(2\pi e|K_{X_1+\cdots+X_k}|^{\frac{1}{n}}) = \frac{n}{2}\log(2\pi e)+\frac{n}{2}\log(k). \quad (16)$$

Using (4), Conjecture 2 tells

$$\frac{n}{2}\log(2\pi e) \le h(X_1)+cn, \quad (17)$$

hence

$$h(X_1+\cdots+X_k) \le h(X_1)+cn+\frac{n}{2}\log(k). \quad (18)$$

We conclude that

$$N(X_1+\cdots+X_k) \le c'kN(X_1), \quad (19)$$

where $c'=e^{2c}$. □

Several reverse entropy power inequalities exist in literature (see, e.g., [12], [6], [2], [11]), but none of them solve Conjecture 3. For example, Bobkov and Madiman [6] established that for all independent log-concave random vectors X, Y in $\mathbb{R}^n$, there exist linear volume-preserving maps u, v such that

$$N(u(X)+v(Y)) \le c(N(X)+N(Y)), \quad (20)$$

where $c>0$ is a universal constant. By induction, one deduces the existence of a universal constant $c>0$ such that for every $k\ge 1$, for all independent log-concave random vectors $X_1,\ldots,X_k$, there exist volume-preserving maps $u_1,\ldots,u_k$ such that

$$N(u_1(X_1)+\cdots+u_k(X_k)) \le c^{k-1}(N(X_1)+\cdots+N(X_k)). \quad (21)$$

Inequality (21) yields a dependence in k which is exponential, while Conjecture 3 asks for a linear dependence in k. However, inequality (21) is valid for independent non-identically distributed log-concave random vectors, while Conjecture 3 is restricted to i.i.d isotropic log-concave random vectors. In the i.i.d. case, a slower growth with k can be demonstrated. For example, the following reverse EPI, due to Cover and Zhang [12] (see also [29], [23]), holds for i.i.d. log-concave random vectors X_i,

$$N(X_1+\cdots+X_k) \le k^2 N(X_1). \quad (22)$$

Currently the best known bound to the hyperplane conjecture, due to Klartag [19] (see also [21]), is equivalent (via Theorem 1) to the following reverse EPI

$$N(X_1+\cdots+X_k) \le c\sqrt{n}\,kN(X_1), \quad (23)$$

which holds for i.i.d. log-concave random vectors X_i, and strengthens (22) when k is large. Conjecture 3 asks whether k^2 in (22) and $c\sqrt{n}\,k$ in (23) can be replaced with ck, for some universal constant $c>0$.

III. A Simple Proof for the Entropic Hyperplane Conjecture for (c_1, c_2)-Regular Random Vectors

In what follows, X denotes an isotropic random vector in $\mathbb{R}^n$ (not necessarily log-concave). Following the terminology of [25], we say that $X \sim f_X$ is (c_1, c_2)-regular if

$$\|\nabla \log f_X(x)\|_2 \le c_1\|x\|_2 + c_2, \quad \forall x\in\mathbb{R}^n, \quad (24)$$

for some non-negative constants $c_1, c_2 \ge 0$. Here, $\|\cdot\|_2$ denotes the Euclidean norm in $\mathbb{R}^n$.

As a straightforward consequence of the EPI, we will show that Conjecture 2 holds whenever X is (c_1, c_2)-regular as long as $c_1 \le c$ and $c_2 \le c\sqrt{n}$ for some universal constant $c>0$.

Proof. It is well known that the EPI implies the following inequality, sometimes referred to as isoperimetric inequality for entropies, which is equivalent to the log-Sobolev inequality,

$$N(X)I(X) \ge N(Z)I(Z) = n, \quad (25)$$

where Z is standard Gaussian (see, e.g., [13]). Here,

$$I(X) \triangleq \mathbb{E}\left[\|\nabla \log f_X(X)\|_2^2\right], \quad (26)$$

denotes the Fisher information of X. Hence,

$$\frac{I(X)}{n} = \frac{I(X)}{I(Z)} \ge \frac{N(Z)}{N(X)} = e^{\frac{2}{n}D(X||Z)}. \quad (27)$$

Taking the logarithm, we deduce that

$$D(X||Z) \le \frac{n}{2}\log\left(\frac{I(X)}{n}\right). \quad (28)$$

In view of (28), to establish (3) it is enough to show that

$$I(X) \le c'n, \quad (29)$$

for some universal constant $c'>0$. But since X is (c_1, c_2)-regular, we have

$$\|\nabla \log f_X(x)\|_2^2 \le c_1^2\|x\|_2^2 + 2c_1c_2\|x\|_2 + c_2^2. \quad (30)$$

Hence,

$$I(X) = \mathbb{E}\left[\|\nabla \log f_X(X)\|_2^2\right] \le c_1^2\mathbb{E}\left[\|X\|_2^2\right] + 2c_1c_2\mathbb{E}\left[\|X\|_2\right] + c_2^2. \quad (31)$$

We have by Jensen inequality and the isotropicity of X,

$$\mathbb{E}\left[\|X\|_2\right] \le \sqrt{\mathbb{E}\left[\|X\|_2^2\right]} = \sqrt{n}. \quad (32)$$

As $c_1 \leq c$ and $c_2 \leq c\sqrt{n}$ for some universal constant $c > 0$ by assumption, we conclude that

$$I(X) \leq c' n, \tag{33}$$

for some universal constant $c' > 0$. □

Remark 1. As shown in [25], random vectors of the form $X + Z_{\sigma^2}$, where $X \perp\!\!\!\perp Z_{\sigma^2} \sim \mathcal{N}(0, \sigma^2 I_n)$, are (c_1, c_2)-regular, with $c_1 = \frac{3}{\sigma^2}$ and $c_2 = \frac{4}{\sigma^2}\mathbb{E}\left[\|X\|_2\right]$. In particular, the entropic hyperplane conjecture holds for the random vector $\frac{X+Z}{\sqrt{2}}$, where Z is standard Gaussian, and X is an isotropic random vector independent of Z.

Remark 2. We do not know whether the assumption of (c_1, c_2)-regularity combined with isotropicity necessarily implies that $c_1 \leq c$ and $c_2 \leq c\sqrt{n}$, for some universal constant $c > 0$.

IV. The entropic hyperplane conjecture for $(c_1\|x\|_p^q + c_2)$-regular log-concave random vectors

Let $p > 1$ and $q > 0$. Following the terminology of [20], we say that a random vector $X \sim f_X$ in $\mathbb{R}^n$ is $(c_1\|x\|_p^q + c_2)$-regular if for almost all $x \in \mathbb{R}^n$,

$$\|\nabla \log f_X(x)\|_2 \leq c_1\|x\|_p^q + c_2, \tag{34}$$

for some non-negative constants $c_1, c_2 \geq 0$. Here,

$$\|x\|_p \triangleq \left(\sum_{i=1}^n |x_i|^p\right)^{\frac{1}{p}}. \tag{35}$$

Note that (c_1, c_2)-regular distributions correspond to $(c_1\|x\|_2 + c_2)$-regular distributions.

A typical example is the distribution with density $Ce^{-\frac{\|x\|_p^q}{q}}$, $p, q > 1$, $C > 0$ is the normalizing constant, which is $\|x\|_{\min\{p,2(p-1)\}}^{q-1}$-regular.

Theorem 2. *Let $p \geq 1$ be independent of the dimension n. The entropic hyperplane conjecture holds for $(c_1\|x\|_p^q + c_2)$-regular isotropic log-concave random vectors in $\mathbb{R}^n$, whenever $q \leq \frac{p}{2}$, $c_1 \leq c$, and $c_2 \leq c\sqrt{n}$, for some universal constant $c > 0$.*

In order to prove Theorem 2, we need the following reverse Jensen inequality valid for log-concave random variables.

Lemma 1 ([18], [7]). *Let X be a centered log-concave random variable. Then, for every $1 \leq q \leq r < +\infty$,*

$$\mathbb{E}\left[|X|^r\right]^{\frac{1}{r}} \leq C(q, r)\mathbb{E}\left[|X|^q\right]^{\frac{1}{q}}, \tag{36}$$

where $C(q, r)$ is a constant depending on q and r only. Moreover, one may take

$$C(q, r) = 2\Gamma(r+1)^{\frac{1}{r}}/\Gamma(q+1)^{\frac{1}{q}}. \tag{37}$$

Proof of Theorem 2. As seen in Section III, it is enough to show that

$$I(X) \leq c' n, \tag{38}$$

for some universal constant $c' > 0$. Since X is $(c_1\|x\|_p^q + c_2)$-regular, we deduce that

$$I(X) \leq c_1^2\mathbb{E}\left[\|X\|_p^{2q}\right] + 2c_1c_2\mathbb{E}\left[\|X\|_p^q\right] + c_2^2. \tag{39}$$

Using Jensen inequality, we have

$$\mathbb{E}\left[\|X\|_p^q\right] \leq \mathbb{E}\left[\|X\|_p^p\right]^{\frac{q}{p}}, \tag{40}$$

$$\mathbb{E}\left[\|X\|_p^{2q}\right] \leq \mathbb{E}\left[\|X\|_p^p\right]^{\frac{2q}{p}}. \tag{41}$$

On the other hand, using the reverse Jensen inequality (36), we have

$$\mathbb{E}\left[\|X\|_p^p\right] = \sum_{i=1}^n \mathbb{E}\left[|X_i|^p\right] \leq C(p)^p \sum_{i=1}^n \mathbb{E}\left[|X_i|^2\right]^{\frac{p}{2}}, \tag{42}$$

for some constant $C(p)$ depending on p only. Since X is isotropic, we have for every $i \in \{1, \ldots, n\}$,

$$\mathbb{E}\left[|X_i|^2\right] = 1. \tag{43}$$

Hence,

$$\mathbb{E}\left[\|X\|_p^p\right] \leq C(p)^p n. \tag{44}$$

As p is independent of n, $2q \leq p$, $c_1 \leq c$, and $c_2 \leq c\sqrt{n}$, for some universal constant $c > 0$, we conclude that

$$I(X) \leq c_1^2 C(p)^{2q} n^{\frac{2q}{p}} + 2c_1c_2C(p)^q n^{\frac{q}{p}} + c_2^2 \leq c' n, \tag{45}$$

for some universal constant $c' > 0$. □

We end this section with a criterion for regularity, which extends [25, Proposition 2] applicable to convolutions of (c_1, c_2)-regular random vectors with Gaussians.

Proposition 1. *Let $V = X + Z$, where $X \perp\!\!\!\perp Z$, $\mathbb{E}\left[\|X\|_r\right] < \infty$, and Z has density*

$$f_Z(z) = c_{r,d}^n \mathrm{e}^{-\frac{\|z\|_r^r}{rd}}, \quad z \in \mathbb{R}^n, \tag{46}$$

with $r \geq 2$, where $d = \mathbb{E}\left[\|Z\|_r^r\right]/n$, and $c_{r,d}$ is the normalizing constant. Then, V is $(c_1\|x\|_r^{r-1} + c_2)$-regular with

$$c_1 = \frac{3 \cdot 2^{r-2}}{d}, \quad c_2 = \frac{2^{r-1}(1 + 2^{r-2})}{d}\mathbb{E}\left[\|X\|_r\right]^{r-1}. \tag{47}$$

Proof. We follow the proof of [25, Proposition 2]. Let f_V be the density of V. One has,

$$\nabla \log f_V(v) = \mathbb{E}\left[\nabla \log f_Z(v - X) | V = v\right]. \tag{48}$$

Hence,

$$\|\nabla \log f_V(v)\|_2 \leq \mathbb{E}\left[\|\nabla \log f_Z(v-X)\|_2 | V=v\right] = \frac{1}{d}\mathbb{E}\left[\|v-X\|_{2(r-1)}^{r-1} | V=v\right]. \quad (49)$$

Since $r \geq 2$, one has $\|v-x\|_{2(r-1)} \leq \|v-x\|_r$. Thus

$$\|\nabla \log f_V(v)\|_2 \leq \frac{1}{d}\mathbb{E}\left[\|v-X\|_r^{r-1} | V=v\right]. \quad (50)$$

Let us denote

$$a(X,v) \triangleq f_Z(v-X)/f_V(v). \quad (51)$$

We have

$$\begin{aligned}\mathbb{E}\left[\|v-X\|_r^{r-1} | V=v\right] &= \mathbb{E}\left[\|v-X\|_r^{r-1} a(X,v)\right] \\ &\leq 2\mathbb{E}\left[\|v-X\|_r^{r-1} 1_{\{a(X,v)\leq 2\}}\right] \\ &\quad + \mathbb{E}\left[\|v-X\|_r^{r-1} a(X,v) 1_{\{a(X,v)>2\}}\right].\end{aligned} \quad (52)$$

Note that

$$\{a(x,v)>2\} = \left\{\|v-x\|_r^r < rd\log\left(\frac{c_{r,d}^n}{2f_V(v)}\right)\right\}. \quad (53)$$

Hence,

$$\begin{aligned}&\mathbb{E}\left[\|v-X\|_r^{r-1} a(X,v) 1_{\{a(X,v)>2\}}\right] \\ &\leq \left[rd\log\left(\frac{c_{r,d}^n}{2f_V(v)}\right)\right]_+^{\frac{r-1}{r}},\end{aligned} \quad (54)$$

where $[x]_+ \triangleq \max\{0,x\}$. We have

$$\begin{aligned}f_V(v) &= \mathbb{E}\left[f_Z(v-X)\right] \\ &\geq \mathbb{E}\left[f_Z(v-X) 1_{\{\|X\|_r \leq 2\mathbb{E}[\|X\|_r]\}}\right] \\ &\geq \mathbb{P}\left[\|X\|_r \leq 2\mathbb{E}\left[\|X\|_r\right]\right] c_{r,d}^n \mathrm{e}^{-\frac{(\|v\|_r + 2\mathbb{E}[\|X\|_r])^r}{rd}} \\ &\geq \frac{c_{r,d}^n}{2}\mathrm{e}^{-\frac{(\|v\|_r + 2\mathbb{E}[\|X\|_r])^r}{rd}},\end{aligned} \quad (55)$$

where the last inequality follows from Markov inequality. We deduce that

$$\begin{aligned}&\mathbb{E}\left[\|v-X\|_r^{r-1} a(X,v) 1_{\{a(X,v)>2\}}\right] \\ &\leq \left(\|v\|_r + 2\mathbb{E}\left[\|X\|_r\right]\right)^{r-1} \\ &\leq 2^{r-2}\left(\|v\|_r^{r-1} + 2^{r-1}\mathbb{E}\left[\|X\|_r\right]^{r-1}\right),\end{aligned}$$

where the last inequality follows from convexity of $x \mapsto x^{r-1}$. We conclude that

$$\begin{aligned}\|\nabla \log f_V(v)\|_2 &\leq \frac{2^{r-1}}{d}\left(\|v\|_r^{r-1} + \mathbb{E}\left[\|X\|_r\right]^{r-1}\right) \\ &\quad + \frac{2^{r-2}}{d}\left(\|v\|_r^{r-1} + 2^{r-1}\mathbb{E}\left[\|X\|_r\right]^{r-1}\right) \\ &= \frac{3\cdot 2^{r-2}}{d}\|v\|_r^{r-1} + \frac{2^{r-1}(1+2^{r-2})}{d}\mathbb{E}\left[\|X\|_r\right]^{r-1}.\end{aligned} \quad (56)$$

□

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