Homework #9 – **Properties of differentiable function, Mean value theorems**

PARTIAL SOLUTIONS

Exercise 1.

Recall that f even means $f(-x) = f(x)$ and f odd means $f(-x) = -f(x)$.

1. Since f is differentiable and even,

$$f'(-x) = \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h\to 0} \frac{f(x-h) - f(x)}{h}$$

$$= -\lim_{h\to 0} \frac{f(x-h) - f(x)}{-h} = -\lim_{H\to 0} \frac{f(x+H) - f(x)}{H} = -f'(x).$$

Exercise 4.

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$

Exercise 5.

$f(x) = 1$ on $[0, 1]$. In this case, we have $f'(x) = 1$ for all $x \in [0, 1]$, and f attains its maximum at 1 and its minimum at 0.

Exercise 6.

$$f(x) = \begin{cases} x^2 \sin^2\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$