Lecture 19: Sections 7.1 and 7.2
System of Equations

Def. A system of equations is a collection of two or more equations, each containing one or more variables.

A solution of a system of equations consists of values of the variables that satisfy each equation in the system. To Solve a system is to find all solutions of the system.

A system is consistent if it has at least one solution; otherwise it is called inconsistent.

Def. A system of the form

\[
\begin{align*}
  a_1x + b_1y &= c_1 \\
  a_2x + b_2y &= c_2
\end{align*}
\]

is a system of two linear equations in two variables.

The solution set of the system is all ordered pairs \((x, y)\) that satisfy the system.
Solving by Graphing

**ex.** Solve each system:

1) \[
\begin{align*}
x + 2y &= 4 \\
x - y &= 1
\end{align*}
\]

2) \[
\begin{align*}
x - y &= -1 \\
-2x + 2y &= 2
\end{align*}
\]

3) \[
\begin{align*}
x + y &= 3 \\
x + y &= -2
\end{align*}
\]

We have three possibilities:

1. The lines intersect at one point. The system is consistent with exactly one solution. The equations are **independent**.

2. The lines coincide. There are infinitely many solutions (every point one the line); the system is consistent and the equations are **dependent**.

3. The lines are parallel. The system is inconsistent, having no solutions.
The Method of Substitution

1. **Solve** for one variable in an equation in terms of the other.

2. **Substitute** the result in the other equation.

3. Solve the resulting equation in one variable.

4. **Back-substitute** into the equation from (1) to find the value of the second variable.

5. Check your solution in **each** equation.

**ex.** Solve the system \[ \begin{aligned} x - 2y &= 1 \\ 4x - 3y &= -1 \end{aligned} \]
The Method of Elimination

Rewrite the equations as necessary so that we can add or subtract two equations to eliminate a variable.

ex. Solve the system:

1) \[
\begin{align*}
3x + 2y &= 4 \\
5x - 2y &= 8
\end{align*}
\]

2) \[
\begin{align*}
\frac{x}{2} - \frac{2y}{3} &= -2 \\
-6x + 8y &= 24
\end{align*}
\]
Nonlinear System of Equations

A **nonlinear system** is a system of equations in which at least one equation is not linear.

**ex.** Solve each of the following systems. Graph each equation and verify the points of intersection.

1. \[
\begin{align*}
y &= x^2 + 1 \\
y &= x + 1
\end{align*}
\]
2) \[
\begin{aligned}
& x^2 + y^2 = 25 \\
& -2x + y = -5
\end{aligned}
\]
3) \[
\begin{aligned}
y &= \sqrt{x} \\
y &= 2 - x
\end{aligned}
\]
Applications

ex. Steve made a profit of $25,000 from the sale of his condominium. He then invested part of that money in a bond fund at 4% interest and the rest in a mutual fund which returns 6% interest. After a year, he had earned $1260. How much did he invest in each account?
ex. An airplane flying into a headwind makes the 1800 miles trip between Phoenix and Pittsburg in 3 hours and 36 minutes. The return trip takes 3 hours. Find the speed of the plane and the speed of the wind, assuming that both remain constant.
ex. A chemist has one solution containing a 10% concentration of acid and a second solution containing a 15% concentration of acid. How many milliliters (ml) of each should be mixed in order to obtain 10 ml of a solution containing a 12% acid concentration?
**ex.** A farmer has $1500 available to build a fence along a straight river to create two identical rectangular pastures. The material for side parallel to the river costs $6 per foot, and materials for the three sides perpendicular to the river costs $5 per foot. Find the dimensions of the pasture if no fence is needed along the river and the total area of the pastures is 6250 ft$^2$. 
Practice.

Solve the system:

1) \[
\begin{align*}
0.4x - 0.2y &= 1 \\
2x &= y + 3
\end{align*}
\]

2) \[
\begin{align*}
5x - 3y &= 16 \\
2x - 4y &= 12
\end{align*}
\]

Answer:
1) \(\emptyset\)
2) \((2, -2)\)