Lecture 21: Section 3.2
Logarithmic Functions

Recall the graph of exponential function $f(x) = a^x$, $a > 1$:

Since $f(x)$ is one-to-one, it has an inverse function.

**Def.** The logarithmic function with base $a$, where $x > 0$, $a > 0$ and $a \neq 1$ is written $f(x) = \log_a x$ and is defined by the relationship

$$y = \log_a x \text{ if and only if }$$

**ex.** Write in exponential form:

1) $\log_3 \left( \frac{1}{9} \right) = -2$

2) $\log_e (3x + 1) = 2$
**ex.** Write in logarithmic form: \(4^{1.5} = 8\)

**ex.** Evaluate:

1) \(\log_2 64 =\)

2) \(\log_3 1 =\)

3) \(\log_3 3 =\)

4) \(\log_{10} \left( \frac{1}{1000} \right) =\)

5) \(\log_{16} 4 =\)

6) \(\log_2 (-1) =\)

7) \(\log_2 0 =\)
Properties of Logarithms

1. Recall: If \( y = \log_a x \), then \( x = \)
   
   That is, \( x > 0 \).

2. \( \log_a 1 = \)

3. \( \log_a a = \)

4. **Inverse Properties:**
   
   \[ \log_a a^x = \quad \text{for all real number } x \]
   \[ a^{\log_a x} = \quad \text{for } x > 0 \]

5. **One-to-One Properties:**
   
   If \( \log_a x = \log_a y \), then

**ex.** Evaluate:

1) \( 2^{\log_2 3\pi} = \)

2) \( \log_5 \frac{1}{5} = \)

**ex.** Solve: \( \log_2 (x - 3) = \log_2 9 \)
Graphs of Logarithmic Functions

**ex.** Sketch $y = 2^x$ and $y = \log_2 x$.

**ex.** Sketch $y = \left(\frac{1}{2}\right)^x$ and $y = \log_{\frac{1}{2}} x$.
Properties of the graph of $f(x) = \log_a x$

Compare $f(x) = \log_a x$ and $f^{-1}(x) = a^x$:

$$f(x) = \log_a x \quad f^{-1}(x) = a^x$$

1. Domain:
2. Range:
3. Intercept:
4. Asymptote:
5. increasing if 
   decreasing if
6. points on 
   the graph

**ex.** Graph $f(x) = \log_3(x + 1)$
The Natural Logarithmic Function

\[ y = \log_e x = \ln x \] if and only if

Note the following:

\[ \ln 1 = \quad \ln e = \]

**Inverse Properties:**

\[ e^{\ln x} = \]

\[ \ln(e^x) = \]

**One-to-One Property:**

If \( \ln x = \ln y \), then

**ex.** Evaluate:

1) \( e^{\ln(2x+3)} = \)

2) \( \ln \left( \frac{1}{e} \right) = \)

**ex.** Solve: \( \ln(x^2 - x) = \ln 6 \)
**ex.** Graph and find the domain and vertical asymptote of $f(x)$:

1) $f(x) = \ln x$

2) $f(x) = \ln(x - 2) + 1$
3) \( f(x) = \ln(-x) + 2 \)

**Common Logarithm Function**

\( y = \log_{10} x = \log x \) if and only if

**ex.** Evaluate:

\[
\log 1 = \log 10 = \\
\log 10000 = \log \frac{1}{\sqrt{10}} = 
\]
Applications

ex. The loudness level of a sound, \( D \), in decibels, is given by

\[
D = 10 \log \left( \frac{I}{10^{-12}} \right),
\]

where \( I \) is the intensity of a sound in watts per square meter.

1) Determine the decibel level of a sound with an intensity of \( 10^{-2} \) watt per square meter.

2) Determine the decibel level of a sound with an intensity of 1 watts per square meter.

3) The intensity of a sound in part (2) is 100 times as great as the intensity in part (1). By how much is the decible level increased?
Practice.

Determine the domain of each function.

1) \( f(x) = \log(x^2 - 2x) \)

2) \( g(x) = \log \left( \frac{3 - x}{x + 3} \right) \)

3) \( h(x) = \ln |x - 2| \)

Answer.

\( (\infty, 0) \cup (2, \infty) \)

\( (-3, 3) \)

\( (\infty, 2) \cup (2, \infty) \)