Lecture 3: Section A.3
Polynomials and Factoring

Polynomials

Def. A polynomial in $x$ is an expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, \cdots, a_0$ are real numbers and $n$ is a nonnegative integer.

NOTE:

1) $a_n, \cdots, a_0$ are the coefficients of each term. If $a_n \neq 0$, the polynomial has degree $n$, and $a_n$ is called the leading coefficient.

2) Polynomials with one, two, and three terms are called monomials, binomials, and trinomials, respectively.

3) In standard form, a polynomial is written with descending powers of $x$.

4) A polynomial with all $a_i = 0$ ($i = 0, 1, \cdots, n$) is called the zero polynomial. No degree is assigned to the zero polynomial.
ex. State whether each is a polynomial. For each polynomial, rewrite it in standard form and find its degree.

1) $-2$

2) $2x^4 - 1 + \frac{3}{x}$

3) $2x^5 + 4x - \sqrt{2}x$

4) $5x - 7 + \sqrt{3}x^8 - 4x^5$

**Evaluate a polynomial:**
Evaluate the polynomial $2x^3 + 3x^2 - 2x + 5$ when $x = -2, 0, 1$

<table>
<thead>
<tr>
<th>x</th>
<th>Substitute</th>
<th>Simplify</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
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**Operations with Polynomials**

1. **Addition and Subtraction** of Polynomials:

Combine like terms (same variable to the same power) by adding their coefficients.
ex. Find the sum:

\[(x^3 + 6x^2 - 4x + 7) + (3x^2 + 2x - 4)\]

ex. Find the difference:

\[(x^3 + 6x^2 - 4x + 7) - (3x^2 + 2x - 4)\]

2. Multiplication of Polynomials:

Use the Properties of Exponents, Commutative, Associative and Distributive Properties to find the product of two polynomials.

ex. \((-3x^8)(6x^5)\)

ex. Find the product: \((3x - 4)(x^3 - 2x - 6)\)

When multiplying two binomials we use **FOIL**.

ex. \((2x - 5)(4x + 7)\)
Special Products

1. Difference of Two Squares

\[(u + v)(u - v) = \]

2. Square of a Binomial

\[(u + v)^2 = \]
\[(u - v)^2 = \]

**NOTE:** \((u + v)^2 \neq u^2 + v^2\)

**ex.** Find \((3x + \sqrt{5})^2\)

**ex.** Find \((x^2 + x + 2)(x^2 + x - 2)\)
3. Cube of a Binomial

\[(u + v)^3 = \]
\[(u - v)^3 = \]

**ex.** Find \((x^2 - 2)^3\)

**Factoring**

\[x^2 - 4 = (x - 2)(x + 2)\]

To **factor** a polynomial is to express it as the **product** of polynomial expressions, called **factors**.

A polynomial which can not be written as the product of factors other than itself and 1 is called **prime**. A polynomial is **factored completely** if it is written as the product of prime polynomials only.
Factoring out the Greatest Common Factor (GCF)

The greatest common factor (gcf) of two integers is the greatest integer that is a factor of two integer.

To find the gcf of two integers you can factor each integer completely and find product of prime factors that are common to both integer.

\[24 = \hspace{1cm} 36 = \]

GCF =

You can use similar methods to find GCF of two monomials:

\[2x^3yz = \hspace{1cm} 6x^2y^2 = \]

GCF =

Use the distributive property

\[ab + ac = a(b + c)\]

to factor out monomial factors common to each term.
ex. Factor completely:

1) \[3x^2 - 6x\]

2) \[8x^4y^2 + 6x^3y^3 - 2xy^4\]

3) \[5(x - 2)^4 - 10(x - 2)^3\]

4) \[y + 6\]
Factoring Special Polynomial Forms

1. Difference of Two Squares
   \[ u^2 - v^2 = \]

2. Perfect Square Trinomial
   \[ u^2 + 2uv + v^2 = \]
   \[ u^2 - 2uv + v^2 = \]

ex. Factor completely:

   1) \((x + 2)^2 - y^2\)

   2) \(16x^4 - 81\)

   3) \(x^2 - 8x + 16\)

   4) \(9y^2 + 30y + 25\)
3. Sum and Difference of Cubes
\[ u^3 + v^3 = \]
\[ u^3 - v^3 = \]

ex. Factor completely:

1) \(27x^3 - 1\)

2) \(y^6 + 8\)

Factoring Trinomial \(x^2 + Bx + C\)
\[ x^2 + Bx + C = (x + a)(x + b) \]
where \(ab = \)
and \(a + b = \)

ex. Factor: \(x^2 - 7x - 30\)
Factoring Trinomial \( Ax^2 + Bx + C, \ A \neq 1 \)

**ex.** Factor: 1) \( 2x^2 - 5x + 2 \)

2) \( 6x^2 - x - 12 \)

**Factoring by Grouping**

We use this method for polynomials with more than 3 terms. Group terms with common factors, and then factor the result.

**ex.** Factor: \( xy - 2y - 4x + 8 \)
ex. Factor a trinomial by grouping: $6x^2 - x - 12$

Practice.

1) Factor completely:

$$(x - 4)^3(4)(3x + 1)^3(3) + (3x + 1)^4(3)(x - 4)^2$$

2) Factor completely: $2y^4 - 8y^2$

3) Find the product $(m + n - 3)(m - n - 3)$

4) Find the product $(x + y)(x - y)(x^2 + y^2)$

5) Find the sum $(5 + 2\sqrt{6})^2 + (5 - 2\sqrt{6})^2$