Def. The rectangular or Cartesian coordinate system is formed by coordinate axes: horizontal $x$-axis and vertical $y$-axis.

The origin is the point of intersection of coordinate axes. The axes divide the coordinate plane or $xy$-plane into four quadrants.

Each point $P$ in the $xy$-plane corresponds to an ordered pair $(x, y)$ of real numbers called the coordinates of $P$. 
To plot \( P = (x, y) \):

\( x \)-coordinate: directed distance of the point from the \( y \)-axis

\( y \)-coordinate: directed distance of the point from the \( x \)-axis

**ex.** Plot \( P_1 = (-3, -2) \) and \( P_2 = (1, 3) \).

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**ex.** Locate all points which are 2 units above the \( x \)-axis. In which quadrant(s), if any, do they lie?

**ex.** Locate all points with \( x \)-coordinate 0.
Pythagorean Theorem

For a right triangle with hypotenuse of length $c$ and sides of lengths $a$ and $b$, we have $a^2 + b^2 = c^2$.

**NOTE:** The converse of the theorem is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Distance Formula

The distance between two points $A = (x_1, y_1)$ and $B = (x_2, y_2)$ is given by

\[ d(A, B) = \]
ex. If \( P = (-2, 4) \) and \( Q = (4, -3) \), find the distance between \( P \) and \( Q \).

Midpoints

Midpoint Formula

The midpoint \( M \) of the line segment with endpoints \( A = (x_1, y_1) \) and \( B = (x_2, y_2) \) is

\[
(x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

ex. Find the midpoint of the line segment that joins \(( -1, 2)\) and \(( 5, 4)\).

ex. The point \((x, -2)\) is the midpoint of the segment with endpoints \((2, -1)\) and \((3, y)\). Find \(x\) and \(y\).
Lecture 7, Part II: Section 1.2
Graph of Equations

An equation in two variables \(x\) and \(y\) is a statement in which two expressions involving \(x\) and \(y\) are equal.

An ordered pair \((a, b)\) is a solution of an equation in two variables if the equation is true when \(a\) is substituted for \(x\) and \(b\) is substituted for \(y\).

The graph of an equation in two variables is the set of points in the \(xy\)-plane which are solutions to the equation.

Graph by Plotting Points:

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth curve or line.
ex. Graph by plotting points:

1) $2x - y = 3$

2) $y = x^2 - 3$
Intercepts

**Def.** The point(s) at which a graph intersects (crosses or touches) the coordinate axes are called **intercepts**.

An $x$-intercept is the $x$-coordinate of a point at which the graph intersects the $x$-axis. To find it,

An $y$-intercept is the $y$-coordinate of a point at which the graph intersects the $y$-axis. To find it,

**ex.** Find the intercepts of the equation and graph: $y - 3 = 2x$
Symmetry

**Def.** A graph is **symmetric with respect to the $x$-axis** if for every point $(x, y)$ on the graph, the point $(x, -y)$ is also on the graph.

![Graph symmetric with respect to the x-axis]

**Algebraic test:** The graph of an equation is symmetric with respect to the $x$-axis if replacing $y$ with $-y$ in the equation yields an equivalent equation.

**Def.** A graph is **symmetric with respect to the $y$-axis** if for every point $(x, y)$ on the graph, the point $(-x, y)$ is also on the graph.

![Graph symmetric with respect to the y-axis]

**Algebraic test:** The graph of an equation is symmetric with respect to the $y$-axis if replacing $x$ with $-x$ in the equation yields an equivalent equation.
Def. A graph is **symmetric with respect to the origin** if for every point \((x, y)\) on the graph, the point \((-x, -y)\) is also on the graph.

![Graph](image.png)

**Algebraic test:** The graph of an equation is symmetric with respect to the origin if replacing \(x\) with \(-x\) and \(y\) with \(-y\) in the equation yields an equivalent equation.

**ex.** Test for symmetry and graph the following.

1) \(y = |x| - 1\)
2) \( x = y^2 - 1 \)

3) \( y = \frac{1}{x} \)
Circles

**Def.** A circle is a set of points in the $xy$-plane that are a fixed distance $r$, called the radius, from a fixed point $(h, k)$, the center of the circle.

Find a relationship between $x$ and $y$ if $P = (x, y)$ is any point on the circle.

**Standard Form of the equation of a circle** with radius $r$ and center $(h, k)$:

**NOTE:** The unit circle is centered at the origin with radius 1. Its equation:
ex. Describe the graph of \( (x + 1)^2 + (y - 2)^2 = 9 \).

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ex. Write the equation of the circle with center \((1, -2)\) and containing the origin. Sketch the circle.
General form of the equation of a circle:

\[ x^2 + y^2 + ax + by + c = 0 \]

**NOTE:** If an equation of a circle is in the general form, we use the method of completing the square to put the equation in standard form so that we can identify its center and radius.

**ex.** Find the center and radius of the circle

\[ 2x^2 + 2y^2 + 8x - 4y + 9 = 0. \]
Practice.

1) Find all points having an $x$-coordinate of 2 whose distance from the point $(5, -1)$ is 5.

2) Show that the points $(1, 0)$, $(-1, 1)$ and $(2, 7)$ are the vertices of a right triangle.

3) Find the equation of the circle that has a diameter with endpoints $(-3, 5)$ and $(5, -1)$.

4) Find the equation of the circle center at $(2, -3)$ and tangent to the $x$-axis (i.e. the circle just touches the $x$-axis).

Answer.

1) $(2, 3)$, $(2, -5)$

2) $L7 - 14$