

EXAM I

A. In pencil, write and encode in the spaces indicated:

- 1) Name (last name, first initial, middle initial)
- 2) UF ID number
- 3) **SKIP** Section number

B. Under “special codes” code in the test ID numbers 1, 1.

- 2 3 4 5 6 7 8 9 0
- 2 3 4 5 6 7 8 9 0

C. At the top right of your answer sheet, for “Test Form Code”, encode A.

- B C D E

D.

- 1) This test consists of 13 multiple choice questions, 6 points each, plus 2 free response questions totaling 28 points. The test is counted out of 100 points.
- 2) The time allowed is 90 minutes.
- 3) You may write on the test.
- 4) Raise your hand if you need more scratch paper or if you have a problem with your test. **DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.**

E. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

F. When you are finished:

- 1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.
- 2) You must turn in your scantron and tearoff sheets to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.
- 3) The answers will be posted in Canvas within one day after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted in Canvas within one week of the exam.

Summary: Formulas for Curves in Space

Position function: $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

Velocity: $\vec{v} = \vec{r}'$

Acceleration: $\vec{a} = \vec{v}' = \vec{r}''$

Unit tangent vector: $\hat{T} = \frac{\vec{v}}{|\vec{v}|}$

Principal unit normal vector: $\hat{N} = \frac{\hat{T}'}{|\hat{T}'|}$

Unit binormal vector: $\hat{B} = \hat{T} \times \hat{N}$

Curvature: $\kappa = \left| \frac{d\hat{T}}{ds} \right| = \frac{|\hat{T}'|}{|\vec{v}|} = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$

Components of acceleration: $\vec{a} = a_T \hat{T} + a_N \hat{N}$

$$\text{with } a_T = \vec{a} \cdot \hat{T} = |\vec{v}|' = \frac{\vec{v} \cdot \vec{a}}{|\vec{v}|}$$

$$\text{and } a_N = \vec{a} \cdot \hat{N} = |\vec{v}| |\hat{T}'| = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|}$$

NOTE: Be sure to bubble the answers for questions 1–13 on your scantron.

Questions 1 – 13 are worth 6 points each.

1. Let $\vec{a} = \langle -2, 3, 2 \rangle$, $\vec{b} = \langle 1, -1, 1 \rangle$, and $\vec{c} = \langle -2, -2, 1 \rangle$. Compute the volume of the parallelepiped formed by $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(a) 19

(b) 10

(c) -10

(d) 11

(e) -11

$$\begin{aligned} V &= \left| \vec{a} \cdot (\vec{b} \times \vec{c}) \right| \\ \vec{b} \times \vec{c} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ -2 & -2 & 1 \end{vmatrix} = \vec{i}(-1+2) - \vec{j}(1+2) + \vec{k}(-2-2) \\ &= \langle 1, -3, -4 \rangle \\ \langle -2, 3, 2 \rangle \cdot \langle 1, -3, -4 \rangle &= -2(1) + 3(-3) + 2(-4) \\ &= -2 - 9 - 8 = -19 \end{aligned}$$

2. Which of the following are true

I. For any vectors $\vec{u}$ and $\vec{v}$, $|\vec{u} + \vec{v}| = |\vec{u}| + |\vec{v}|$. **X** try two random vectors.

II. For any vectors $\vec{u}$ and $\vec{v}$, $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$ **✓** because multiplication and addition are commutative

III. For any vectors $\vec{u}$ and $\vec{v}$, $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$ **X** $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$

IV. For any vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$, $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$ **✓** addition is commutative

(a) I and II

(b) II and IV

(c) II, III, IV

(d) All of the above

(e) None of the above

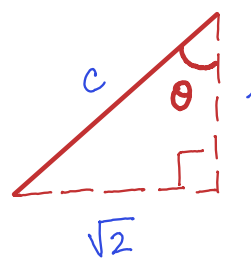
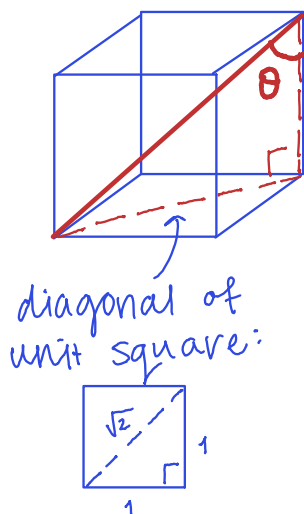
3. Which of the following is the angle between the (big) diagonal of a unit cube and one of its edges, where the diagonal and the edge start at the same point?

(a) $\arcsin \frac{1}{\sqrt{3}}$

(b) $\arccos \frac{1}{\sqrt{3}}$

(c) $\arcsin \frac{2}{\sqrt{6}}$

(d) $\arccos \frac{2}{\sqrt{6}}$



$$a^2 + b^2 = c^2$$

$$2 + 1 = c^2$$

$$\sqrt{3} = c$$

SOHCAHTOA

$$\cos(\theta) = \frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

4. Find an equation of the set of all points equidistant from the points $(1, 2, 0)$ and $(-1, 0, 0)$.

(a) $4x - 4y = 4$

(b) $4x + y = 4$

(c) $x + y = 1$

(d) $x + y = 4$

(e) $x + 4y = 4$

Let D be the distance from (x, y, z) to $(1, 2, 0)$

Let d be the distance from (x, y, z) to $(-1, 0, 0)$

$$D = \sqrt{(1-x)^2 + (2-y)^2 + (0-z)^2}$$

$$d = \sqrt{(-1-x)^2 + (0-y)^2 + (0-z)^2}$$

Want $D = d$:

$$\sqrt{(1-x)^2 + (2-y)^2 + z^2} = \sqrt{(-1-x)^2 + y^2 + z^2}$$

$$1 - 2x + \cancel{x^2} + 4 - 4y + \cancel{y^2} + \cancel{z^2} = 1 + 2x + \cancel{x^2} + y^2 + \cancel{z^2}$$

$$4 = 4x + 4y$$

$$1 = x + y$$

5. Find the distance between the center of the sphere $x^2 - 6x + z^2 = -2y - y^2$ and the point $(0, 9, -3)$.

(a) $\sqrt{109}$

(b) $9\sqrt{5}$

(c) $\sqrt{118}$

(d) $5\sqrt{10}$

$$\begin{aligned} x^2 - 6x + y^2 + 2y + z^2 &= 0 \\ x^2 - 6x + (3)^2 + y^2 + 2y + (1)^2 + z^2 &= 9 + 1 \\ (x-3)^2 + (y+1)^2 + z^2 &= 10 \\ \text{center } (3, -1, 0) \\ d &= \sqrt{(3-0)^2 + (-1-9)^2 + (0+3)^2} \\ &= \sqrt{9 + 100 + 9} = \sqrt{118} \end{aligned}$$

6. Find the parametric equations for the tangent line to the curve

$$x = t^3, y = t^2, z = t$$

at the point $(-1, 1, -1)$.

$$\begin{aligned} \text{happens at } t &= -1 \quad \uparrow \\ r(t) &= \langle t^3, t^2, t \rangle \\ r'(t) &= \langle 3t^2, 2t, 1 \rangle \\ r'(-1) &= \langle 3, -2, 1 \rangle \end{aligned}$$

(a) $x = -1 + 3t, y = 1 - 2t, z = -1 + t$

(b) $x = -1 + 3t^2, y = 1 + 2t, z = -1 + t$

(c) $x = -1 - 3t, y = 1 + 2t, z = 1 - t$

(d) $x = -1 + 3t, y = 1 - 2t, z = 1 + t$

(e) None of the above

$$\begin{cases} x = 3t - 1 \\ y = -2t + 1 \\ z = t - 1 \end{cases}$$

7. A force $\vec{F} = 8\hat{i} - 6\hat{j} + 9\hat{k}$ moves an object from the point $(0,10,8)$ to the point $(6,12,20)$. Find the work done on the object.

$$\begin{aligned} F &= \langle 8, -6, 9 \rangle & d &= \langle 6-0, 12-10, 20-8 \rangle \\ & & &= \langle 6, 2, 12 \rangle \end{aligned}$$

(a) 108

(b) -48

(c) 9

(d) 144

(e) 126

$$\begin{aligned} W &= F \cdot d \\ &= \langle 8, -6, 9 \rangle \cdot \langle 6, 2, 12 \rangle \\ &= 8(6) - 6(2) + 9(12) \\ &= 48 - 12 + 108 \\ &= 144 \end{aligned}$$

8. The space curve $\vec{r}(t) = \langle \underbrace{4t^2}_x, \underbrace{2t \sin(t)}_y, \underbrace{t \cos(t)}_z \rangle$ lies on the _____.

(a) cone $\frac{x^2}{4} = \frac{y^2}{4} + z^2$

(b) cone $\frac{x^2}{4} = y^2 + \frac{z^2}{4}$

(c) paraboloid $x = y^2 + 4z^2$

(d) paraboloid $x = y^2 + z^2$

(e) paraboloid $x = 4y^2 + z^2$

$$\begin{aligned} y^2 + 4z^2 &= 4t^2 \sin^2(t) + 4t^2 \cos^2(t) \\ &= 4t^2 (\sin^2 + \cos^2) \\ &= 4t^2 = x \end{aligned}$$

$$y^2 + 4z^2 = x$$

9. Find the equation of the intersection of the planes $x + y + z = 1$ and $-x - 2z = 4$.

$$\vec{n}_1 = \langle 1, 1, 1 \rangle \quad \vec{n}_2 = \langle -1, 0, -2 \rangle$$

(a) $\vec{r}(t) = \langle 2t - 6, 2t + 6, t + 1 \rangle$

(b) $\vec{r}(t) = \langle -2t - 6, 2t + 6, t + 1 \rangle$

(c) $\vec{r}(t) = \langle -2t - 6, t + 6, 2t + 1 \rangle$

(d) $\vec{r}(t) = \langle -2t - 6, 3t + 6, t + 1 \rangle$

(e) $\vec{r}(t) = \langle -2t - 6, t + 6, t + 1 \rangle$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ -1 & 0 & -2 \end{vmatrix}$$

$$= \mathbf{i}(-2-0) - \mathbf{j}(-2+1) + \mathbf{k}(0+1)$$

$$= \langle -2, 1, 1 \rangle$$

note $(-6, 6, 1)$ is a point on both planes

$$\text{line: } \begin{cases} x = -2t - 6 \\ y = t + 6 \\ z = t + 1 \end{cases}$$

10. Compute the scalar projection of $\langle 2, 0, 4 \rangle$ onto $\langle 4, 0, 0 \rangle$.

(a) 2

$$\text{comp}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$$

(b) 8

$$\vec{u} \cdot \vec{v} = 2(4) + 0(0) + 4(0) = 8$$

(c) -2

$$|\vec{v}| = \sqrt{16 + 0 + 0} = 4$$

(d) 4

$$\text{comp}_{\vec{v}} \vec{u} = \frac{8}{4} = 2$$

(e) $\frac{1}{2}$

11. Find the position vector of a particle with acceleration $\vec{a}(t) = \langle 0, 0, 2 \rangle$ and initial conditions $\vec{v}(0) = \langle 1, 2, -1 \rangle$ and $\vec{r}(0) = \langle -1, 1, 3 \rangle$.

(a) $\langle t, -2t, t^2 + 3t \rangle$

$$\vec{v}(t) = \int \vec{a}(t) = \int \langle 0, 0, 2 \rangle dt$$

$$= \langle a, b, 2t + c \rangle$$

(b) $\langle t - 1, 2t + 1, t^2 - t + 3 \rangle$

$$\vec{v}(0) = \langle 1, 2, -1 \rangle = \langle a, b, c \rangle$$

$$\vec{v}(t) = \langle 1, 2, 2t - 1 \rangle$$

(c) $\langle t^2 + 2, -2t, t^3 + 2t^2 + 3t + 2 \rangle$

$$\vec{r}(t) = \int \vec{v}(t) = \int \langle 1, 2, 2t - 1 \rangle dt$$

(d) $\langle t + 2, 2t, t^2 - 3t + 2 \rangle$

$$= \langle t + d, 2t + e, t^2 - t + f \rangle$$

$$\vec{r}(0) = \langle -1, 1, 3 \rangle = \langle d, e, f \rangle$$

(e) None of the above

$$\vec{r}(t) = \langle t - 1, 2t + 1, t^2 - t + 3 \rangle$$

12. Find the distance between the point $(1, 2, 3)$ and the plane $x + y + z = 1$.

$$(x_0, y_0, z_0) \quad ax + by + cz + d = 0$$

(a) $\frac{1}{\sqrt{2}}$

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

(b) $\frac{5}{\sqrt{3}}$

$$= \frac{|1 + 2 + 3 - 1|}{\sqrt{3}} = \frac{5}{\sqrt{3}}$$

(c) $\frac{2}{\sqrt{3}}$

(d) $\frac{2}{\sqrt{2}}$

(e) $\frac{1}{\sqrt{6}}$

13. Find the area of the parallelogram determined by vectors $\vec{a}$ and $\vec{b}$, where

$$|\vec{a}| = 7, \quad |\vec{b}| = 1, \quad \vec{a} \cdot \vec{b} = 3.$$

$$A = |\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

(a) $\frac{\sqrt{10}}{3}$

(b) $\frac{2\sqrt{10}}{7}$

(c) $14\sqrt{10}$

(d) $2\sqrt{10}$

(e) $\sqrt{10}$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$3 = 7 \cos \theta$$

$$\frac{3}{7} = \cos \theta \rightarrow \text{need } \sin \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta + \frac{9}{49} = 1$$

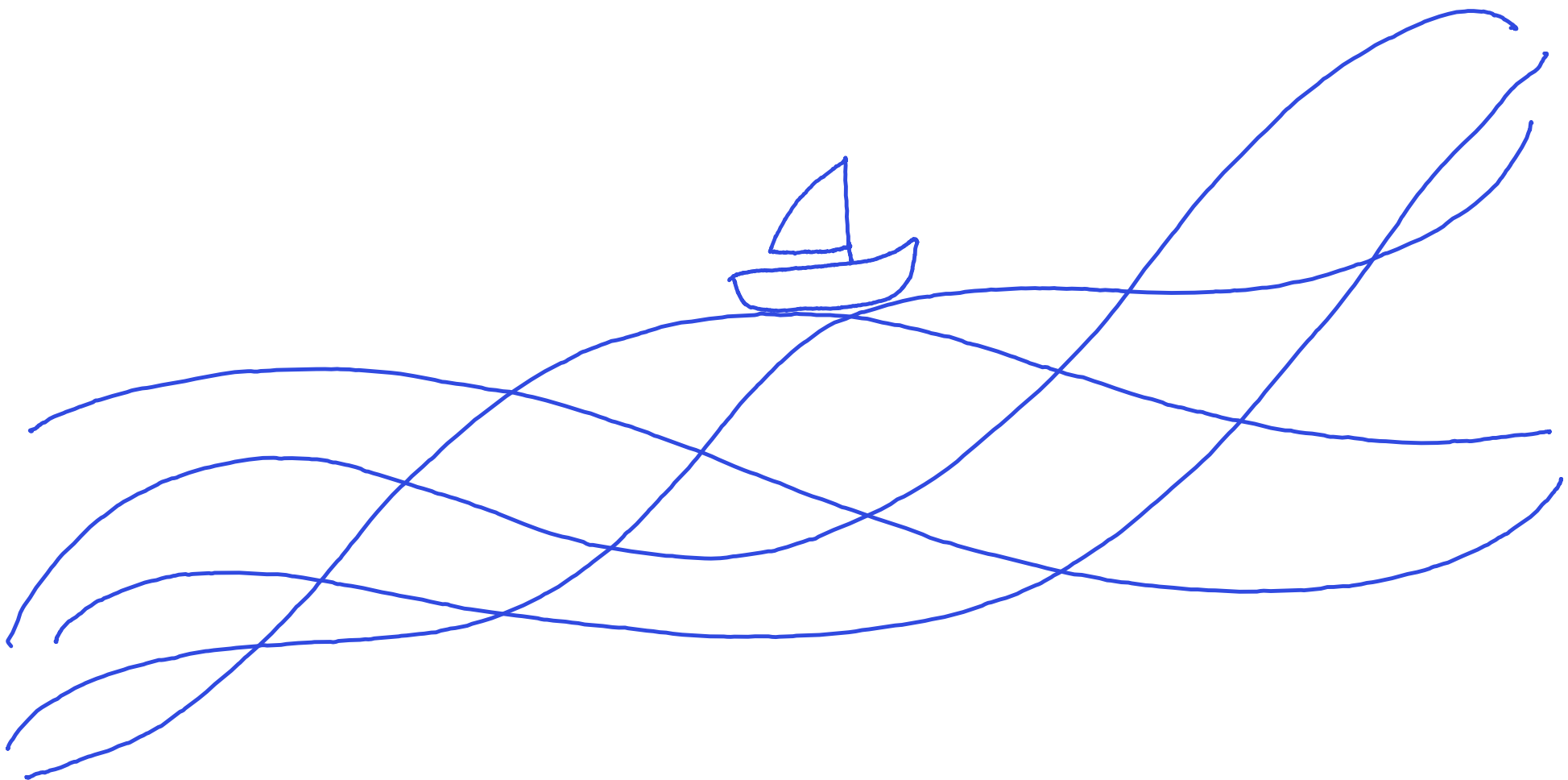
$$\sin^2 \theta = \frac{40}{49}$$

$$\sin \theta = \frac{\sqrt{40}}{7} = \frac{2\sqrt{10}}{7}$$

$$A = |\vec{a}| |\vec{b}| \sin \theta$$

$$= 7 \cdot \frac{2\sqrt{10}}{7} = 2\sqrt{10}$$

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MAC 2313 Exam I, Part II Free Response

Name: _____ Discussion Period _____

Circle your TA's Name

Lodewyk Jansen Van Rensburg	Jasen Lai	Christian Austin	Kushagri Sharma
Theodosios Alvalis	Dylan Connell	Abby Owens	Dmitrii Smorchkov
Umesha Wijerathne	Vincent Holmlund	Xiaochun Duan	Harris Barton
Michael Waite	Teresa Castano	Jun Cho	Alexander Wong

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. Let C be the curve given by $\vec{r}(t) = \langle 2t, \ln t, t^2 \rangle$.

(a) (6 points) Find the length of the curve for $1 \leq t \leq 4$.

$$L = \int_a^b |\vec{r}'(t)| dt \quad \vec{r}'(t) = \langle 2, \frac{1}{t}, 2t \rangle$$

$$L = \int_1^4 \sqrt{4 + \frac{1}{t^2} + 4t^2} dt = \int_1^4 \sqrt{\frac{4t^2 + 1 + 4t^4}{t^2}} dt$$

$$= \int_1^4 \sqrt{\frac{(2t^2 + 1)^2}{t^2}} dt = \int_1^4 \frac{2t^2 + 1}{t} dt = \int_1^4 2t + \frac{1}{t} dt$$

$$= t^2 + \ln(t) \Big|_1^4 = 16 + \ln(4) - 1 - \ln(1) = \boxed{15 + \ln(4)}$$

(b) (6 points) Find the curvature at $t = 1$.

$$\kappa = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$$

$$\vec{v}(t) = \langle 2, \frac{1}{t}, 2t \rangle$$

$$\vec{a}(t) = \langle 0, -t^{-2}, 2 \rangle$$

$$|\vec{v}| = \frac{2t^2 + 1}{t} \text{ from (a)}$$

$$|\vec{v}(1)|^3 = (3)^3 = 27$$

$$\vec{v}(1) = \langle 2, 1, 2 \rangle$$

$$\vec{a}(1) = \langle 0, -1, 2 \rangle$$

$$\vec{v} \times \vec{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 2 \\ 0 & -1 & 2 \end{vmatrix} = \mathbf{i}(2+2) - \mathbf{j}(4-0) + \mathbf{k}(-2-0)$$

$$= \langle 4, -4, -2 \rangle$$

$$|\vec{v} \times \vec{a}|(1) = \sqrt{16+16+4} = \sqrt{36} = 6$$

$$\kappa = \frac{6}{27} = \frac{2}{9}$$

2. Let T be the triangle with vertices $A = (1, 2, 3)$, $B = (3, 3, 6)$, and $C = (2, 2, 5)$.

(a) (5 points) Compute $\vec{AB} \times \vec{AC}$

$$\vec{AB} = \langle 2, 1, 3 \rangle \quad \vec{AC} = \langle 1, 0, 2 \rangle$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 3 \\ 1 & 0 & 2 \end{vmatrix} = \mathbf{i}(2-0) - \mathbf{j}(4-3) + \mathbf{k}(0-1) \\ = \langle 2, -1, -1 \rangle$$

(b) (6 points) Find the equation of the plane containing the triangle T .

$$\vec{AB} \times \vec{AC} = \vec{n} = \langle 2, -1, -1 \rangle$$

$$\langle 2, -1, -1 \rangle \cdot \langle x-1, y-2, z-3 \rangle = 0 \quad (\text{point } A)$$

$$2(x-1) - (y-2) - (z-3) = 0$$

$$2x - 2 - y + 2 - z + 3 = 0$$

$$2x - y - z = -3$$

(c) (5 points) Find the area of the triangle T .

$$A = \frac{1}{2} |\vec{AB} \times \vec{AC}| \\ = \frac{1}{2} \sqrt{4 + 1 + 1} = \frac{\sqrt{6}}{2}$$