

Name: _____

Key

Quiz 4 (L14-L16)

MAC2313 Summer 2026

1. [4 pts] Calculate $\frac{\partial z}{\partial t}$ where $z = 4x^2 + 3y^2$, $x = \cos(t)$, $y = \sin(t)$.

$$\begin{aligned}\frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} \\ &= 8x(-\sin(t)) + 6y(\cos(t)) \\ &= -8\cos(t)\sin(t) + 6\cos(t)\sin(t) \\ &= -2\cos(t)\sin(t)\end{aligned}$$

2. [6 pts] Find the directional derivative of the function $f(x, y) = x^2y + y^3$ at the point $(2, 2)$ in the direction of the vector $\vec{v} = \langle 2, 1 \rangle$.

$$D_{\hat{v}}f(a, b) = \nabla f(a, b) \cdot \hat{v}$$

$$\nabla f = \langle 2xy, x^2 + 3y^2 \rangle \quad \nabla f(2, 2) = \langle 8, 16 \rangle$$

$$|\hat{v}| = \sqrt{4 + 1} = \sqrt{5} \quad \hat{v} = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$$

$$D_{\hat{v}}f(2, 2) = \langle 8, 16 \rangle \cdot \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$$

$$= 8\left(\frac{2}{\sqrt{5}}\right) + 16\left(\frac{1}{\sqrt{5}}\right)$$

$$= \frac{16}{\sqrt{5}} + \frac{16}{\sqrt{5}} = \frac{32}{\sqrt{5}}$$