RELATED RATES:

1. Determine what you are solving for/what you are trying to find.
2. Write down the "given" quantities in the problem (draw a picture).
3. Write an equation relating the given quantities.
4. Take derivative implicitly (of both sides) with respect to time.
5. Plug in (given) quantities and solve.

LOGARITHMIC DIFFERENTIATION:

1. \( \frac{d}{dx} \ln(x) = \frac{1}{x} \)
2. \( \frac{d}{dx} \log_a x = \frac{1}{\ln(a)x} \)
3. \( \frac{d}{dx} \left[ \ln(f(x)) \right] = \frac{f'(x)}{f(x)} \)
4. \( \frac{d}{dx} \left[ \log_a (f(x)) \right] = \frac{f'(x)}{f(x)\ln(a)} \)

LINEARIZATION:

\( L(x) = f'(a)(x-a) + f(a) \)

\( L(x) \) IS THE LINEAR APPROXIMATION TO \( f(x) \) AT \( x = a \).

WE USE THE ABOVE EQUATION TO APPROXIMATE \( f(x) \), SO \( f(x) \approx L(x) \).

*THE ERROR IS GIVEN BY \( |f(x) - L(x)| \).

DIFFERENTIALS:

A DIFFERENTIAL IS A SMALL CHANGE IN A VARIABLE.

1. \( dx \) IS THE DIFFERENTIAL FOR \( x \), \( dy \) IS THE DIFFERENTIAL FOR \( y \).
(2) $\Delta x = dx$ \Rightarrow "Actual" Change in $x$

(3) For $y = f(x)$, $dy = f'(x) \, dx$ and $\Delta y \approx dy$

(4) The actual change in $y$ is $\Delta y = f(x + \Delta x) - f(x)$

(5) Relative Error is $\frac{\Delta y}{y_0} \approx \frac{dy}{y_0}$

(6) The equation for linear approximation is now given by:

$L(x) = f(a) + dy \Rightarrow "New = Old + Change"$