**Interval on Which a Function is Increasing or Decreasing:**

If \( f(x) \) is increasing if \( f'(x) > 0 \)

If \( f(x) \) is decreasing if \( f'(x) < 0 \)

**Concavity and Point of Inflection:**

If \( f' \) is increasing over an interval \( I \), then \( f \) is concave up over \( I \).

If \( f' \) is decreasing over an interval \( I \), then \( f \) is concave down over \( I \).

* If \( f''(x) > 0 \) for all \( x \in [a, b] \) \( \Rightarrow \) \( f \) is concave up over \( [a, b] \)

* If \( f''(x) < 0 \) for all \( x \in [a, b] \) \( \Rightarrow \) \( f \) is concave down over \( [a, b] \).

If \( f \) is continuous at a point \( x \) and if \( f \) changes concavity at \( x \), then \( (x, f(x)) \) is an inflection point of \( f \).

**To Find Inflection Points:**

1. Determine where \( f \) changes concavity
2. Evaluate \( f \) at the values found in 1

**L'Hospital's Rule:**

Suppose that \( f \) and \( g \) are differentiable and \( g'(x) \neq 0 \) near \( a \). Suppose that either

1. \( \lim_{x \to a} f(x) = 0 \) and \( \lim_{x \to a} g(x) = 0 \)

or

2. \( \lim_{x \to a} f(x) = \pm \infty \) and \( \lim_{x \to a} g(x) = \pm \infty \)

Then, \( \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \) if it exists, or is \( \pm \infty \).

This is an indeterminate form \( 0/0 \) or \( \infty/\infty \).

**L'Hospital's Rule Applied to Indeterminate Products:**
IF \( \lim_{x \to a} f(x) = 0 \) AND \( \lim_{x \to a} g(x) = \pm \infty \), THEN TO FIND \( \lim_{x \to a} f(x)g(x) \), WE FIND

\[ \lim_{x \to a} \frac{g(x)}{f(x)} \] (THIS IS OK BECAUSE \( g(x) = g(x) \cdot \frac{f(x)}{f(x)} \))

↑

NOW, THIS IS INDETERMINANT FORM \( \frac{0}{\infty} \)

L'HÔPITAL'S RULE APPLIED TO INDETERMINANT DIFFERENCES:

IF \( \lim_{x \to a} f(x) = \infty \) AND \( \lim_{x \to a} g(x) = \infty \), THEN TO FIND \( \lim_{x \to a} f(x) - g(x) \), REWRITE \( f(x) - g(x) \)

AS A SINGLE FRACTION, AND THEN APPLY L'HÔPITAL'S RULE.

L'HÔPITAL'S RULE APPLIED TO INDETERMINANT POWERS:

WANT TO FIND \( \lim_{x \to a} f(x)^{g(x)} \), BUT WE OBTAIN THE INDETERMINANT FORM \( \infty^0 \), \( 0^0 \)

OR \( \infty^\infty \). THEN, FOLLOW THE STEPS BELOW TO FIND THE LIMIT:

1. **LET** \( y = f(x)^{g(x)} \)

2. **TAKE** \( \ln \) **OF BOTH SIDES** AND **USE PROPERTIES** **OF \( \ln \) TO SIMPLIFY:**

\[ \ln(y) = \ln(f(x)^{g(x)}) = g(x) \ln(f(x)) \]

3. **TAKE THE LIMIT AS** \( x \to a \) **OF BOTH SIDES:**

\[ \lim_{x \to a} \ln(y) = \lim_{x \to a} \{g(x) \ln(f(x))\} \]

4. **USE AN APPROPRIATE APPLICATION OF L'HÔPITAL'S RULE TO FIND THE LIMIT OF**

\[ \lim_{x \to a} \{g(x) \ln(f(x))\} \]

SAID YOU FIND THAT \( \lim_{x \to a} \{g(x) \ln(f(x))\} = L \) \( (L \) \( CAN \) \( BE \) \( \infty \)\)

5. **SINCE** \( \lim_{x \to a} \{g(x) \ln(f(x))\} = L \), WE HAVE \( \lim_{x \to a} \ln(y) = L \).
(6) Since \( \ln \) is continuous, rewrite as follows:

\[
\lim_{x \to a} \ln(y) = L \iff \ln \left( \lim_{x \to a} y \right) = L
\]

(7) Solve \( \ln \left( \lim_{x \to a} y \right) = L \) as follows:

\[
\ln \left( \lim_{x \to a} y \right) = L \\
\Rightarrow e^{\ln \left( \lim_{x \to a} y \right)} = e^L \\
\Rightarrow \lim_{x \to a} y = e^L
\]

Then, since \( y = f(x)^{g(x)} \), we have

\[
\lim_{x \to a} f(x)^{g(x)} = e^L
\]