1. Find the domain and the range of the function

\[ f(x, y, z) = \ln\left(\sqrt{4 - 2x^2 - y^2 - z^2}\right) \]

\[ \text{domain is given by } 4 - 2x^2 - y^2 - z^2 > 0 \]

\[ 2x^2 + y^2 + z^2 < 4 \]

\[ \left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 + \left(\frac{z}{\sqrt{2}}\right)^2 < 1 \]

in order for \( \sqrt{ } \) to make sense, the inside should be \( > 0 \), but it cannot be \( 0 \), since \( \ln(0) \) does not accept \( 0 \)

\[ \text{range: as inside } \to 0, \text{ output values } \to -\infty \]

the maximum value of output is \( \ln(\sqrt{4}) = \ln 2 \).

\[ (-\infty, \ln 2] \]

2. Does \( \lim_{(x,y) \to (0,0)} \frac{x \sqrt{y}}{x^2 + y^{2/3}} \) exist? Justify your answer.

No: consider the limit from path \( x = 0 \) and \( y = x^3 \)

\[ \lim_{(x,y) \to (0,0)} \frac{x \sqrt{y}}{x^2 + y^{2/3}} = \lim_{(x,y) \to (0,0)} \frac{0 \sqrt{y}}{0 + y^{2/3}} = 0 \]

\[ \lim_{(x,y) \to (0,0)} \frac{x \sqrt{y}}{x^2 + y^{2/3}} = \lim_{(x,y) \to (0,0)} \frac{x^3 \sqrt{x^3}}{x^2 + (x^3)^{2/3}} = \lim_{(x,y) \to (0,0)} \frac{x^2}{2x^2} = \frac{1}{2} \]

the limit through 2 different paths has two different values. Hence the limit DNE.
3. Show that

\[ \lim_{(x,y) \to (0,0)} \frac{x^2y}{x^2 + y^2} = 0. \]

**Method 1:** Use polar coordinates

\[
x = r \cos \theta \\
y = r \sin \theta
\]

\[
\lim_{(r \to 0)} \frac{r^3 \cos^2 \theta \sin \theta}{r^2} = \lim_{r \to 0} r \cos^2 \theta \sin \theta = 0
\]

as \( r \to 0 \) for any value of \( \theta \)

**Method 2:** Use Squeeze Theorem.

\[ \frac{x^2}{x^2 + y^2} \leq 1 \quad \text{always, (why?)} \]

\[ -y \leq \frac{x^2y}{x^2 + y^2} \leq y \]

as \( y \to 0 \), \( \text{LHS} \to 0 \\	ext{RHS} \to 0 \]

hence by Squeeze Theorem

\[ \frac{x^2y}{x^2 + y^2} \to 0 \]

4. Characterize all level surfaces of the function \( f(x, y, z) = x^2 + y^2 - z^2 \).

\( f(x, y, z) = 0 \) \( \Rightarrow \) \( x^2 + y^2 - z^2 = 0 \), gives a cone.

\( f(x, y, z) > 0 \) \( \Rightarrow \) \( x^2 + y^2 = z^2 + C \) hyperboloid of one sheet

say \( \text{Const C} \)

\( f(x, y, z) < 0 \) \( \Rightarrow \) \( x^2 + y^2 = z^2 - C \) hyperboloid of two sheets.

\( \text{Const} - C, \text{C} \)

5. If the limit of a function \( f(x, y, z) \) exists at the point \( (a, b, c) \), then \( f \) is continuous at \( (a, b, c) \).

(a) TRUE if \( f \) is continuous, then limit exists, the converse is false.

(b) FALSE

6. The range of the function is the set of all real values that can be plugged-in to the function.

(a) TRUE it's the set of all output values.

(b) FALSE