MHF 3202, Dr. Block, Problem Set 1, due 3-23-2020

1. Suppose that t is a real number. Prove that there exists a real number w such that $\frac{w+1}{w-2} = t$ if and only if $t \neq 1$.

   Hint: Here is an outline of a proof:

   Suppose that t is a real number.

   First, we prove that if there exists a real number w such that $\frac{w+1}{w-2} = t$, then $t \neq 1$. By way of contradiction, suppose that there is some real number w such that $\frac{w+1}{w-2} = t$, and $t = 1$.

   You fill in this part. You must prove something that gives a contradiction.

   This is a contradiction. We conclude that if there exists a real number w such that $\frac{w+1}{w-2} = t$, then $t \neq 1$.

   Second, we prove that if $t \neq 1$, then there exists a real number w such that $\frac{w+1}{w-2} = t$. Suppose that $t \neq 1$. Set $w =??$.

   You must figure out what to set w equal to. This is scratch work. Of course the w may depend on the t.

   Then you must prove each of the following.

   w is a well-defined real number.

   $w \neq 2$.

   $\frac{w+1}{w-2} = t$.

   This completes the proof.

2. Prove that for every $\epsilon > 0$ there exists $\delta > 0$ such that if $x \in \mathbb{R}$ and $|x-3| < \delta$, then $|x^2 - 5x + 6| < \epsilon$.

   Hint: Here is an outline of a proof:

   Suppose that $\epsilon > 0$. Set $\delta =??$.

   Suppose that $x \in \mathbb{R}$ and $|x-3| < \delta$.

   **Finish the proof by showing that $|x^2 - 5x + 6| < \epsilon$.

   **A key part of the proof involves scratch work to find a suitable δ.
   **