1. Suppose that $A$, $B$, and $C$ are sets. Prove that

$$(A \cup C) \subseteq (B \cup C)$$

if and only if

$$(A - C) \subseteq (B - C).$$

Proof:
First, suppose that $A \cup C \subseteq B \cup C$. Suppose that $x \in A - C$. Then $x \in A$ and $x \notin C$. Since $x \in A$, it follows that $x \in A \cup C$. Hence, $x \in B \cup C$. Since $x \in B \cup C$ and $x \notin C$ we have $x \in B$. Thus, $x \in B - C$. Since $x$ was arbitrary we conclude that $A - C \subseteq B - C$.

Second, suppose that $A - C \subseteq B - C$. Suppose that $x \in A \cup C$. Suppose that $x \notin C$. Then $x \in A - C$. It follows that $x \in B - C$. Hence, $x \in B$. We conclude that if $x \notin C$, then $x \in B$.

It follows that $x \in B \cup C$. Since $x$ was arbitrary, we conclude that $A \cup C \subseteq B \cup C$.

2. Suppose that $A$ and $B$ are sets. Prove that if

$$\mathcal{P}(A \cup B) \subseteq (\mathcal{P}(A) \cup \mathcal{P}(B)),$$

then either $A \subseteq B$ or $B \subseteq A$.

Proof:
We prove the contrapositive. Suppose that $A$ is not a subset of $B$, and $B$ is not a subset of $A$. There exists $x \in A - B$ and $y \in B - A$. Set $D = \{x, y\}$. Then $D \in \mathcal{P}(A \cup B)$, but $D \notin (\mathcal{P}(A) \cup \mathcal{P}(B))$. It follows that $\mathcal{P}(A \cup B)$ is not a subset of $(\mathcal{P}(A) \cup \mathcal{P}(B))$. 