\[ \exists z \in \mathbb{R} \quad \forall x \in \mathbb{R}^+ \quad [\exists y \in \mathbb{R} \, (y - x = \frac{y}{x}) \iff x \neq z] \]

**Proof:** Let \( z = 1 \). Suppose \( x \in \mathbb{R}^+ \).

First, suppose \( \exists y \in \mathbb{R} \) \( (y - x = \frac{y}{x}) \). Proceeding by contradiction, suppose \( x = z \). Then \( y - 1 = y \). This contradicts the fact that \( y - 1 < y \). Hence, \( x \neq z \).

Second, suppose \( x \neq z \). Set \( y = \frac{x^2}{x-1} \). Then \( y \) is a well-defined real number as \( x \neq 1 \). We have

\[ y - x = \frac{x^2}{x-1} - x = \frac{x^2 - x(x-1)}{x-1} = \frac{x}{x-1} \]

and \( \frac{y}{x} = \frac{\left(\frac{x^2}{x-1}\right)}{x} = \frac{x^2}{x(x-1)} = \frac{x}{x-1} = y - x \).

\[ \square \]