1. (10 points) Prove the following theorem. If \( f : [a, b] \to \mathbb{R} \) is continuous, then \( f \) is Riemann integrable on \([a, b]\).

2. (10 points) Let \( f : [-2, 3] \to \mathbb{R} \) be defined by
\[
f(x) = \begin{cases} 
2|x| + 1 & \text{if } x \text{ is rational} \\
0 & \text{if } x \text{ is irrational}
\end{cases}
\]
Prove that \( f \) is not Riemann integrable on \([-2, 3]\).

3. (8 points) Evaluate the given limit.
\[
\lim_{x \to 0^+} \arctan(x \ln x)
\]

4. (8 points) Evaluate the given limit.
\[
\lim_{x \to -\infty} \frac{\sqrt{x^2 + 1}}{x}
\]

5. (8 points) Evaluate the given limit.
\[
\lim_{x \to \infty} (e^x + x)^{\frac{1}{x}}
\]

6. (2 points) Determine if the statement is true or false.
If \( f : [0, 1] \to \mathbb{R} \) is continuous, then there exists \( c \in (0, 1) \) such that \( f(c) = \int_0^1 f \).

7. (2 points) Determine if the statement is true or false.
If \( f : [0, 1] \to [0, 1] \) is Riemann integrable and \( g : [0, 1] \to \mathbb{R} \) is Riemann integrable, then the composition \( g \circ f : [0, 1] \to \mathbb{R} \) is Riemann integrable.