Advanced Calculus I, Dr. Block, Sample Final Exam, Fall 2019

There are seven problems worth a total of 50 points. Justify your answer in each problem.

1. (7 points) Determine where the given function is continuous.

$$f(x) = \begin{cases} \exp(\frac{1}{x}) \text{ if } x < 0 \\ \sin x \text{ if } x \geq 0 \end{cases}$$

2. (7 points) Locate and classify all of the points of discontinuity of the given function.

$$f(x) = \begin{cases} 2x \text{ if } x = \dfrac{1}{n}, \text{ and } n \in \mathbb{N} \\ 1 \text{ otherwise} \end{cases}$$

Note: Recall that $\mathbb{N} = \{1, 2, 3, \dots\}$.

3. (7 points) Locate and classify all of the points of discontinuity of the given function.

$$f(x) = \begin{cases} x \sin \dfrac{1}{x} \text{ if } x < 0 \\ \cos(\pi x) \text{ if } 0 \leq x \leq 1 \\ x \text{ if } x > 1 \end{cases}$$

4. (7 points) Determine if f is differentiable at the indicated point.

$$f(x) = \begin{cases} (\sin x)^2 \text{ if } x \leq 0 \\ x - \sin x \text{ if } x > 0 \end{cases}$$

$$\text{at } x = 0.$$

5. (7 points) Determine where the given function is differentiable.

$$f(x) = \begin{cases} x^2 + x - 1 \text{ if } x \text{ is rational} \\ x^3 \text{ if } x \text{ is irrational} \end{cases}$$

6. (8 points) Prove the following theorem:

Suppose that $D \subset \mathbb{R}$, and $f : D \to \mathbb{R}$. Suppose that f has a relative minimum at $c \in (a, b) \subset D$. If f is differentiable at $x = c$, then $f'(c) = 0$.

7. (7 points) Use the inverse function theorem to evaluate $\frac{d}{dx} \arcsin x$.