Advanced Calculus I, Dr. Block, Sample Final Exam, Fall 2019

There are seven problems worth a total of 50 points. Justify your answer in each problem.

1. (7 points) Determine where the given function is continuous.

\[ f(x) = \begin{cases} \exp\left(\frac{1}{x}\right) & \text{if } x < 0 \\ \sin x & \text{if } x \geq 0 \end{cases} \]

2. (7 points) Locate and classify all of the points of discontinuity of the given function.

\[ f(x) = \begin{cases} 2x & \text{if } x = \frac{1}{n}, \text{ and } n \in \mathbb{N} \\ 1 & \text{otherwise} \end{cases} \]

Note: Recall that \( \mathbb{N} = \{1, 2, 3, \ldots\} \).

3. (7 points) Locate and classify all of the points of discontinuity of the given function.

\[ f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x < 0 \\ \cos(\pi x) & \text{if } 0 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases} \]

4. (7 points) Determine if \( f \) is differentiable at the indicated point.

\[ f(x) = \begin{cases} (\sin x)^2 & \text{if } x \leq 0 \\ x - \sin x & \text{if } x > 0 \end{cases} \]

at \( x = 0 \).

5. (7 points) Determine where the given function is differentiable.

\[ f(x) = \begin{cases} x^2 + x - 1 & \text{if } x \text{ is rational} \\ x^3 & \text{if } x \text{ is irrational} \end{cases} \]

6. (8 points) Prove the following theorem:

Suppose that \( D \subset \mathbb{R} \), and \( f : D \to \mathbb{R} \). Suppose that \( f \) has a relative minimum at \( c \in (a, b) \subset D \). If \( f \) is differentiable at \( x = c \), then \( f'(c) = 0 \).

7. (7 points) Use the inverse function theorem to evaluate \( \frac{d}{dx} \arcsin x \).