ADVANCED CALCULUS I, DR. BLOCK, SAMPLE EXAM 2, FALL 2019

There are seven problems worth a total of 50 points.

1. (10 points) Prove that the following sequence $\{a_n\}$ converges to 0 using only the definition (without using any theorems). Show your scratch work and the formal proof.

$$a_n = \frac{n+9}{n^2+5}$$

2. (10 points) Consider two sequences $\{a_n\}$ and $\{b_n\}$, where the sequence $\{a_n\}$ diverges to infinity and the sequence $\{a_n\, b_n\}$ converges. Prove that the sequence $\{b_n\}$ must converge to zero.

3. (9 points) Determine whether the given sequence converges, diverges to ∞, diverges to $-\infty$, or oscillates. If the sequence converges, find the limit. Justify your answer, using appropriate theorems.

$$a_n = \frac{(-2)^n}{n^3}$$

4. (9 points) Determine whether the given sequence converges, diverges to ∞, diverges to $-\infty$, or oscillates. If the sequence converges, find the limit. Justify your answer, using appropriate theorems.

$$a_n = \frac{(\sin n)(\sqrt[n]{5n^4})}{n}$$

5. (3 points) Determine if the statement is true or false. If $\{a_n\}$ is the sequence given by $a_n = \sin\frac{n\pi}{4}$, and $\{b_n\}$ is the sequence given by $b_n = \frac{\sqrt{2}}{2}$, then $\{b_n\}$ is a subsequence of $\{a_n\}$.

6. (3 points) Determine if the statement is true or false. If $\{a_n\}$ is the sequence given by $a_n = (-1)^n\frac{2n+3}{n+7}$, then $\liminf_{n\to\infty} a_n = -2$.

7. (3 points) Determine if the statement is true or false. If S is the range of the sequence $\{a_n\}$ given by $a_n = \sin\frac{n\pi}{2}$, then 0 is an accumulation point of S.

8. (3 points) Determine if the statement is true or false. If S is the set of all rational numbers, then every real number is an accumulation point of S.