ADVANCED CALCULUS I, DR. BLOCK, SAMPLE EXAM 3, FALL 2019

1. (4 points) Complete the following definition: Suppose that $f: D \to \mathbb{R}$, where D is a subset of \mathbb{R} . Suppose that $L \in \mathbb{R}$ and a is an accumulation point of $D \cap (a, \infty)$. We say that $\lim_{x \to a^+} f(x) = L$ if and only if

2. (4 points) Complete the following definition: Suppose that $f: D \to \mathbb{R}$, where D is a subset of \mathbb{R} . Suppose that a is an accumulation point of D. We say that $\lim_{x\to a} f(x) = -\infty$ if and only if

3. (10 points) Evaluate the given limit. Show your work and justify your answer.

$$\lim_{x \to 0^+} \left[(\exp(\frac{1}{x}) + \sin(\frac{1}{x})) \right]$$

4. (10 points) Evaluate the given limit. Show your work and justify your answer.

$$\lim_{x \to 0^-} x \sqrt{\frac{7}{x^2} - 5}$$

5. (10 points) Locate and classify all of the points of discontinuity. Justify your answer.

$$f(x) = \begin{cases} x \text{ if } x = \pm \frac{1}{n}, \ n \in \mathbb{N} \\ x^2 \text{ otherwise} \end{cases}$$

Recall that $\mathbb{N} = \{1, 2, 3, \dots\}$

6. (4 points) Determine if the statement is true or false.

If D is a finite subset of \mathbb{R} , then every function $f: D \to \mathbb{R}$ is continuous.

7. (4 points) Determine if the statement is true or false.

The function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \frac{\sin(\sin x)}{x} & \text{if } x \neq 0\\ 0 & \text{if } x = 0 \end{cases}$$

is continuous.

8. (4 points) Determine if the statement is true or false.

If $f : \mathbb{R} \to \mathbb{R}$ and the sequence $\{f(n)\}, n \in \mathbb{N}$ converges to a real number L, then $\lim_{x\to\infty} f(x) = L$.