## ADVANCED CALCULUS I, DR. BLOCK, SAMPLE EXAM 3, FALL 2019

1. (4 points) Complete the following definition: Suppose that $f: D \rightarrow \mathbb{R}$, where $D$ is a subset of $\mathbb{R}$. Suppose that $L \in \mathbb{R}$ and $a$ is an accumulation point of $D \cap(a, \infty)$. We say that $\lim _{x \rightarrow a^{+}} f(x)=L$ if and only if
2. (4 points) Complete the following definition: Suppose that $f: D \rightarrow \mathbb{R}$, where $D$ is a subset of $\mathbb{R}$. Suppose that $a$ is an accumulation point of $D$. We say that $\lim _{x \rightarrow a} f(x)=-\infty$ if and only if
3. (10 points) Evaluate the given limit. Show your work and justify your answer.

$$
\lim _{x \rightarrow 0^{+}}\left[\left(\exp \left(\frac{1}{x}\right)+\sin \left(\frac{1}{x}\right)\right]\right.
$$

4. (10 points) Evaluate the given limit. Show your work and justify your answer.

$$
\lim _{x \rightarrow 0^{-}} x \sqrt{\frac{7}{x^{2}}-5}
$$

5. (10 points) Locate and classify all of the points of discontinuity. Justify your answer.

$$
f(x)=\left\{\begin{array}{l}
x \text { if } x= \pm \frac{1}{n}, n \in \mathbb{N} \\
x^{2} \text { otherwise }
\end{array}\right.
$$

Recall that $\mathbb{N}=\{1,2,3, \ldots\}$
6. (4 points) Determine if the statement is true or false.

If $D$ is a finite subset of $\mathbb{R}$, then every function $f: D \rightarrow \mathbb{R}$ is continuous.
7. (4 points) Determine if the statement is true or false.

The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$
f(x)=\left\{\begin{array}{l}
\frac{\sin (\sin x)}{x} \text { if } x \neq 0 \\
0 \text { if } x=0
\end{array}\right.
$$

is continuous.
8. (4 points) Determine if the statement is true or false.

If $f: \mathbb{R} \rightarrow \mathbb{R}$ and the sequence $\{f(n)\}, n \in \mathbb{N}$ converges to a real number $L$, then $\lim _{x \rightarrow \infty} f(x)=L$.

