ADVANCED CALCULUS I, DR. BLOCK,
SAMPLE EXAM 3 WITH ANSWERS, FALL 2019

1. (4 points) Complete the following definition: Suppose that $f : D \to \mathbb{R}$, where D is a subset of $\mathbb{R}$. Suppose that $L \in \mathbb{R}$ and a is an accumulation point of $D \cap (a, \infty)$. We say that $lim_{x \to a^+} f(x) = L$ if and only if

Answer: for every $\epsilon > 0$ there exists $\delta > 0$ such that $|f(x) - L| < \epsilon$ for all $x \in D$ with $x \in (N_\delta^-(a) \cap (a, \infty))$.

2. (4 points) Complete the following definition: Suppose that $f : D \to \mathbb{R}$, where D is a subset of $\mathbb{R}$. Suppose that a is an accumulation point of D. We say that $lim_{x \to a} f(x) = -\infty$ if and only if

Answer: for every $B < 0$ there exists $\delta > 0$ such that $f(x) < B$ for all $x \in D$ with $x \in N_\delta^-(a)$.

3. (10 points) Evaluate the given limit. Show your work and justify your answer.

$$\lim_{x \to 0^+} \left[(\exp(\frac{1}{x}) + \sin(\frac{1}{x}) \right]$$

Answer:

$$\lim_{x \to 0^+} \exp(\frac{1}{x}) = \lim_{t \to \infty} e^t = \infty.$$

Also, the function $\sin(\frac{1}{x})$ is bounded below. Thus,

$$\lim_{x \to 0^+} \left[(\exp(\frac{1}{x}) + \sin(\frac{1}{x}) \right] = \infty.$$

4. (10 points) Evaluate the given limit. Show your work and justify your answer.

$$\lim_{x \to 0^-} x \sqrt{\frac{7}{x^2} - 5}$$

Answer: Using the fact that for all $x < 0$, $x = -\sqrt{x^2}$, we have

$$\lim_{x \to 0^-} x \sqrt{\frac{7}{x^2} - 5} = \lim_{x \to 0^-} -\sqrt{7 - 5x^2} = -\sqrt{7}.$$

5. (10 points) Locate and classify all of the points of discontinuity. Justify your answer.

$$f(x) = \begin{cases} x \text{ if } x = \pm\dfrac{1}{n},\ n \in \mathbb{N} \\ x^2 \text{ otherwise} \end{cases}$$

Recall that $\mathbb{N} = \{1, 2, 3, \dots\}$

Answer: Each real number $a = \pm\frac{1}{n}$ except for $a = 1$ is a point of discontinuity, and each of these discontinuities is removable. This is the case because for these values of a we have $\lim_{x\to a} f(x) = a^2$ while $f(a) = a$ and $a \neq a^2$. There are no other discontinuites.

6. (4 points) Determine if the statement is true or false.

If D is a finite subset of $\mathbb{R}$, then every function $f : D \to \mathbb{R}$ is continuous.

Answer: True.

7. (4 points) Determine if the statement is true or false.

The function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \dfrac{\sin(\sin x)}{x} \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$$

is continuous.

Answer: False.

8. (4 points) Determine if the statement is true or false.

If $f : \mathbb{R} \to \mathbb{R}$ and the sequence $\{f(n)\}, n \in \mathbb{N}$ converges to a real number L, then $\lim_{x\to\infty} f(x) = L$.

Answer: False.