Advanced Calculus, Dr. Block, Quiz 3 (with solutions), Fall 2019

1. (4 points) Consider the sequences $\{a_n\}$ and $\{b_n\}$, where the sequence $\{a_n\}$ converges to zero. Is it true that the sequence $\{a_n \cdot b_n\}$ must converge to zero. Explain.

   Solution: It is not true that the sequence $\{a_n \cdot b_n\}$ must converge to zero. For example, if $a_n = \frac{1}{n}$ and $b_n = n^2$, then the sequence $\{a_n \cdot b_n\}$ diverges to ∞.

2. (3 points) Determine if the given limit exists, and evaluate the limit if the limit exists. Justify your answer.

$$\lim_{n\to\infty} n \sin \frac{1}{2n}.$$

   Solution: We have

$$n \sin \frac{1}{2n} = \frac{\sin \frac{1}{2n}}{\frac{1}{n}} = \frac{1}{2} \cdot \frac{\sin \frac{1}{2n}}{\frac{1}{2n}}.$$

   It follows from one of the special limits in the Chapter 3 notes (item 13, part 6), that

$$\lim_{n\to\infty} n \sin \frac{1}{2n} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

3. (3 points) Determine whether the given sequence $\{a_n\}$ converges, diverges to ∞, diverges to $-\infty$, or oscillates. Find the limit if the sequence converges. Justify your answer.

$$a_n = \frac{b^n}{n!},$$

   where b is a positive constant.

   Solution: We use the Ratio Test. We have

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty} \frac{b^{n+1} \cdot n!}{(n+1)! \cdot b^n} = \lim_{n\to\infty} \frac{b}{n+1} = 0 < 1.$$

It follows from the Ratio Test that the sequence $\{a_n\}$ converges to zero.