There are 7 problems worth a total of 50 points.

1. (10 points) Use mathematical induction to prove the given statement.
   For every positive integer $n$,
   \[
   \sum_{k=1}^{n} (2k - 1) = n^2.
   \]

2. (8 points) Negate the statement: There exists a real number $b$ such that $f(x) \leq b$ for all $x \in D$.

3. (10 points) Find all real values of $x$ that satisfy the given expression. Express your answer as an interval on the real line, a union of intervals, a finite set of real numbers, or the empty set. Show your work.
   \[|2x - 5| \leq |x + 4|.
   \]

4. (10 points) Prove the following: If $|f(x)| \leq M$ for all $x \in [a, b]$, then
   \[-2M \leq f(x_1) - f(x_2) \leq 2M
   \]
   for any $x_1, x_2 \in [a, b]$.

5. (4 points) Determine if the statement is true or false.
   If $A$ and $B$ are sets, then
   \[(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).
   \]

6. (4 points) Determine if the statement is true or false.
   If $f : X \to Y$ and $A \subseteq X$, then
   \[f^{-1}(f(A)) = A.
   \]

7. (4 points) Determine if the statement is true or false.
   If $S \subseteq \mathbb{R}$ and $k$ is the supremum of $S$, then $k \in S$. 
