ADVANCED CALCULUS I, DR. BLOCK, SAMPLE EXAM 1, FALL 2019

There are 7 problems worth a total of 50 points.

1. (10 points) Use mathematical induction to prove the given statement.

For every positive integer n,

$$\sum_{k=1}^{n}(2k-1) = n^2.$$

2. (8 points) Negate the statement: There exists a real number b such that $f(x) \leq b$ for all $x \in D$.

3. (10 points) Find all real values of x that satisy the given expression. Express your answer as an interval on the real line, a union of intervals, a finite set of real numbers, or the empty set. Show your work.

$$|2x-5| \leq |x+4|.$$

4. (10 points) Prove the following: If $|f(x)| \leq M$ for all $x \in [a, b]$, then

$$-2M \leq f(x_1) - f(x_2) \leq 2M$$

for any $x_1, x_2 \in [a, b]$.

5. (4 points) Determine if the statement is true or false.

If A and B are sets, then

$$(A \backslash B) \cup (B \backslash A) = (A \cup B) \backslash (A \cap B).$$

6. (4 points). Determine if the statement is true or false.

If $f : X \to Y$ and $A \subseteq X$, then

$$f^{-1}(f(A)) = A.$$

7. (4 points). Determine if the statement is true or false.

If $S \subseteq \mathbb{R}$ and k is the supremum of S, then $k \in S$.