

**ADVANCED CALCULUS II, DR. BLOCK,
SPRING 2020, SAMPLE EXAM 2**

Note: There are five problems worth 10 points each.

1. For the function $f : [0, 5] \rightarrow \mathbb{R}$ given by $f(x) = x^2 - 4x$ and the partition $P = \{0, 2, 3, 4, 5\}$ compute the lower sum $L(P, f)$.

2. Suppose that a function $f : [a, b] \rightarrow \mathbb{R}$ is continuous and nonnegative. Prove that if $\int_a^b f = 0$, then $f(x) = 0$ for all $x \in [a, b]$.

3. Prove the following theorem. If $f : [a, b] \rightarrow \mathbb{R}$ is monotonically increasing, then f is Riemann integrable on $[a, b]$.

4. Let $f : [-2, 3] \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 2|x| + 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Prove that f is not Riemann integrable on $[-2, 3]$.

5. Prove that if $f \in R[a, b]$, then $|f| \in R[a, b]$ and

$$\left| \int_a^b f \right| \leq \int_a^b |f|.$$