1. (10 points) Suppose that the function $f$ satisfies $|f(x) - f(t)| \leq (x - t)^2$ for all $x, t \in \mathbb{R}$. Prove that $f$ must be a constant function.

2. (10 points) State and prove Rolle’s Theorem.

3. (8 points) Find the $n$th Taylor polynomial for the given function centered about $x = a$. Show your work and justify your answer.

$$f(x) = \cos x, \quad a = 0.$$ 

**Answer:**

If $n$ is even and $n = 2k$, we have

$$P_n(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^k \frac{x^{2k}}{(2k)!}.$$ 

If $n$ is odd and $n = 2k + 1$, we have

$$P_n(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^k \frac{x^{2k}}{(2k)!}.$$ 

4. (8 points) Evaluate the limit. Show your work and justify your answer.

$$\lim_{x \to \infty} \left( \frac{x}{x + 1} \right)^x$$ 

**Answer:**

$$\frac{1}{e}$$ 

5. (8 points) For the function $f : [0, 5] \to \mathbb{R}$ given by $f(x) = x^2 - 3x$ and the partition $P = \{0, 2, 3, 4, 5\}$ compute the lower sum $L(P, f)$.

**Answer:**

$$(-\frac{9}{4})(2) + (-2)(1) + (0)(1) + (4)(1)$$
6. (2 points) Determine if the statement is true or false.
If \( f(x) = x^3 - 4x + 7 \), then \( f \) is strictly decreasing on the interval \([-1, 1]\).

Answer: True

7. (2 points) Determine if the statement is true or false.
If \( f \) is a bounded function on the interval \([a, b]\) and \( P \) and \( Q \) are partitions of \([a, b]\), then \( L(Q, f) \leq L(P, f) \).

Answer: False