

**ADVANCED CALCULUS II, DR. BLOCK,  
SAMPLE EXAM 1, SPRING 2020**

Note: The maximum possible score is 50.

1. (10 points) State and prove Rolle's Theorem.
2. (10 points) Suppose that the function  $f$  satisfies  $|f(x) - f(t)| \leq (x - t)^2$  for all  $x, t \in \mathbb{R}$ . Prove that  $f$  must be a constant function.
3. (10 points) Find the  $n$ th Taylor polynomial for the given function centered about  $x = a$ . Show your work.

$$f(x) = \cos x, a = 0.$$

4. (10 points) Evaluate the limit. Show your work and justify your answer.

$$\lim_{x \rightarrow \infty} \left( \frac{x}{x+1} \right)^x$$

5. (10 points) If  $f(x) = |x|^3$ , compute  $f'''(0)$ , if possible. Show your work and justify your answer.