then we could use modus ponens to reach our goal. So let’s try treating \( a \in B \)

as our goal and see if that makes the problem easier:

\[
\begin{align*}
\text{Given} & : & \text{Goal} \\
A \subseteq B & & a \in B \\
a \in A & & \\
a \in B \rightarrow a \in C
\end{align*}
\]

Now it is clear how to reach the goal. Since \( a \in A \) and \( A \subseteq B \), \( a \in B \).

\textbf{Solution}

\textbf{Theorem.} Suppose that \( A \subseteq B \), \( a \in A \), and \( a \notin B \setminus C \). Then \( a \in C \).

\textit{Proof.} Since \( a \in A \) and \( A \subseteq B \), we can conclude that \( a \in B \). But \( a \notin B \setminus C \),

so it follows that \( a \in C \).

\textbf{Exercises}

*1. This problem could be solved by using truth tables, but don’t do it that way. Instead, use the methods for writing proofs discussed so far in this chapter. (See Example 3.2.4.)

(a) Suppose \( P \rightarrow Q \) and \( Q \rightarrow R \) are both true. Prove that \( P \rightarrow R \) is true.

(b) Suppose \( \neg R \rightarrow (P \rightarrow \neg Q) \) is true. Prove that \( P \rightarrow (Q \rightarrow R) \) is true.

2. This problem could be solved by using truth tables, but don’t do it that way. Instead, use the methods for writing proofs discussed so far in this chapter. (See Example 3.2.4.)

(a) Suppose \( P \rightarrow Q \) and \( R \rightarrow \neg Q \) are both true. Prove that \( P \rightarrow \neg R \) is true.

(b) Suppose that \( P \) is true. Prove that \( Q \rightarrow \neg(Q \rightarrow \neg P) \) is true.

3. Suppose \( A \subseteq C \), and \( B \) and \( C \) are disjoint. Prove that if \( x \in A \) then \( x \notin B \).

4. Suppose that \( A \setminus B \) is disjoint from \( C \) and \( x \in A \). Prove that if \( x \in C \) then \( x \in B \).

*5. Use the method of proof by contradiction to prove the theorem in Example 3.2.1.

6. Use the method of proof by contradiction to prove the theorem in Example 3.2.5.

7. Suppose that \( y + x = 2y - x \), and \( x \) and \( y \) are not both zero. Prove that \( y \neq 0 \).
8. Suppose that $a$ and $b$ are nonzero real numbers. Prove that if $a < 1/a < b < 1/b$ then $a < -1$.

9. Suppose that $x$ and $y$ are real numbers. Prove that if $x^2 y = 2x + y$, then if $y \neq 0$ then $x \neq 0$.

10. Suppose that $x$ and $y$ are real numbers. Prove that if $x \neq 0$, then if $y = \frac{3x^2 + 2x}{3x^2 + 2}$ then $y = 3$.

11. Consider the following incorrect theorem:

**Incorrect Theorem.** Suppose $x$ and $y$ are real numbers and $x + y = 10$. Then $x \neq 3$ and $y \neq 8$.

(a) What's wrong with the following proof of the theorem?

Proof. Suppose the conclusion of the theorem is false. Then $x = 3$ and $y = 8$. But then $x + y = 11$, which contradicts the given information that $x + y = 10$. Therefore the conclusion must be true.

(b) Show that the theorem is incorrect by finding a counterexample.

12. Consider the following incorrect theorem:

**Incorrect Theorem.** Suppose that $A \subseteq C$, $B \subseteq C$, and $x \in A$. Then $x \in B$.

(a) What's wrong with the following proof of the theorem?

Proof. Suppose that $x \notin B$. Since $x \in A$ and $A \subseteq C$, $x \in C$. Since $x \notin B$ and $B \subseteq C$, $x \notin C$. But now we have proven both $x \in C$ and $x \notin C$, so we have reached a contradiction. Therefore $x \in B$.

(b) Show that the theorem is incorrect by finding a counterexample.

13. Use truth tables to show that modus tollens is a valid rule of inference.

14. Use truth tables to check the correctness of the theorem in Example 3.2.4.

15. Use truth tables to check the correctness of the statements in exercise 1.

16. Use truth tables to check the correctness of the statements in exercise 2.

17. Can the proof in Example 3.2.2 be modified to prove that if $x^2 + y = 13$ and $x \neq 3$ then $y \neq 4$? Explain.