Suppose that $\mathcal{F}$ and $\mathcal{G}$ are nonempty families of sets, and every element of $\mathcal{F}$ is a subset of every element of $\mathcal{G}$. Prove that

$$\bigcup \mathcal{F} \subseteq \bigcap \mathcal{G}.$$ 

Proof: Suppose that $x \in \bigcup \mathcal{F}$. Suppose that $B \in \mathcal{G}$. We can choose $A \in \mathcal{F}$ with $x \in A$. It follows from our hypothesis that $A \subseteq B$. Thus, $x \in B$.

Since $B$ was arbitrary, we conclude that $x \in \bigcap \mathcal{G}$. Since $x$ was arbitrary, we conclude that

$$\bigcup \mathcal{F} \subseteq \bigcap \mathcal{G}.$$