Suppose that $X$ is a nonempty set. Let $A$ denote the set of functions from $X$ to $X$. Let $B$ denote the set of $h \in A$ such that $h$ is bijective. Define a relation $R$ on $A$ by

$$R = \{(f, g) \in A \times A : \exists h \in B \text{ such that } h \circ f = g \circ h\}. $$

Observe that if $(f, g) \in A \times A$, $h \in B$ and $h \circ f = g \circ h$, then

$$h^{-1} \circ h \circ f = h^{-1} \circ g \circ h.$$ 

1. Prove that $R$ is an equivalence relation on $A$.

2. Prove that if $(f, g) \in R$, then $(f \circ f, g \circ g) \in R$.

3. Suppose that $(f, g) \in R$. Suppose that there is some $x \in X$ with $f(x) = x$. Prove that there is some $y \in X$ with $g(y) = y$. 
