1. (5 points) Write the following set by listing the elements of the set between braces.
\( \mathcal{P} \{1, 2\} \times \mathcal{P} \{3\} \)

Answer:
\[ \{ (\varnothing, \varnothing), (\varnothing, \{3\}), (\{1\}, \varnothing), (\{1\}, \{3\}), (\{2\}, \varnothing), (\{2\}, \{3\}), (\{1, 2\}, \varnothing), (\{1, 2\}, \{3\}) \} \]

2. (5 points) Construct a truth table for the formula \( (P \land \sim Q) \rightarrow R \).

Answer:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>\sim Q</th>
<th>P \land \sim Q</th>
<th>(P \land \sim Q) \rightarrow R</th>
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3. (5 points) Negate the following sentence.

For every \( \varepsilon > 0 \) there exists \( \delta > 0 \) such that if \( |x - 5| < \delta \), then \( |x^2 - 25| < \varepsilon \).

Answer: There exists \( \varepsilon > 0 \) such that for all \( \delta > 0 \) there exists \( x \) with \( |x - 5| < \delta \) and \( |x^2 - 25| \geq \varepsilon \).

4. (2 points) Determine whether the following statement is true or false.

\[ \exists a \in \mathbb{Z}, \forall b \in \mathbb{Z}, a + b = 0 \]

Answer: False.
5. (6 points) Prove the following using a direct proof.
If \( x \) is an odd integer, then \( x^2 - 7 \) is even.

Answer: Suppose that \( x \) is an odd integer. Then for some integer \( k \) we have \( x = 2k + 1 \). It follows that
\[
x^2 - 7 = 4k^2 + 4k + 1 - 7 = 4k^2 + 4k - 6 = 2j
\]
where \( j = 2k^2 + 2k - 3 \in \mathbb{Z} \). Therefore, \( x^2 - 7 \) is even.

6. (6 points) Prove the following with contrapositive proof.
Suppose \( a \in \mathbb{Z} \). If \( a^2 \) is not divisible by 4, then \( a \) is odd.

Answer: Suppose that \( a \) is not odd. Then \( a \) is even. So for some integer \( k \) we have \( a = 2k \). It follows that
\[
a^2 = 4k^2 = 4j,
\]
where \( j = k^2 \in \mathbb{Z} \). Therefore, \( a^2 \) is divisible by 4.

7. (6 points) Prove the following using either direct proof or contrapositive proof.
If \( a \in \mathbb{Z} \) and \( a \equiv 1 \pmod{5} \), then \( a^2 \equiv 1 \pmod{5} \).

Answer: (direct proof) Suppose that \( a \in \mathbb{Z} \) and \( a \equiv 1 \pmod{5} \). By definition, \( 5|(a-1) \). So for some integer \( k \) we have \( a - 1 = 5k \). It follows that
\[
a^2 - 1 = (a + 1)(a - 1) = (a + 1)(5k) = 5j,
\]
where \( j = a + k \in \mathbb{Z} \). Thus, \( 5|(a^2 - 1) \). Therefore, \( a^2 \equiv 1 \pmod{5} \).